SUPPLEMENT

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CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1967

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

LINE PLANT PRACTICE C, 1967 (*continued*)

Q. **10.** *A beam of cross section shown in Fig.* 1 *is made of steel. What will be the ratio of its flexural strength compared with that of a beam of the same weight but having a rectangular cross-section with its depth being twice the breadth? Find the maximum stress produced in the I section beam if a load of* 15 *ton is spread uniformly over a simple supported span of* 14 *ft.*

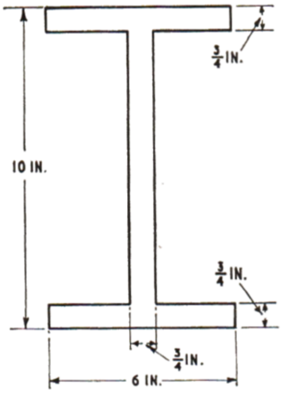


Fig. 1

A. 10. Area of I beam $= (2 \times \frac{3}{4} \times 6) + (8\frac{1}{2} \times \frac{3}{4})$,

$= 15\frac{3}{8}$ in^2.

Areas of I beam and rectangular beam are equal.

Thus if x inches is the width of the rectangular beam

$$\text{Area} = 2x^2.$$

Thus, $$2x^2 = 15\tfrac{3}{8}.$$

$$\therefore \quad \underline{x = \frac{\sqrt{123}}{4} \text{ in.}}$$

For a beam, the flexural strength or modulus of section is given by I/y, where I is the moment of inertia and y is the depth of the neutral axis.

Now, for rectangular beam,

$$\text{flexural strength} = \frac{I}{y} = \frac{bd^3}{12}\Big/\frac{d}{2} = \frac{bd^2}{6}.$$

and, $$b = x\ d = 2x.$$

$$\therefore \frac{I}{y} = \frac{4x^3}{6} = \frac{2x^3}{3} = \frac{2}{3}\left(\frac{\sqrt{123}}{4}\right)^3.$$

$$= 14{\cdot}22 \text{ in.}^3$$

$$\text{Now for the I beam, } I = \frac{6 \times 10^3}{12} - \frac{5\frac{1}{4} \times (8\frac{1}{2})^3}{12},$$

$$= 231{\cdot}5 \text{ in}^4.$$

$$\therefore \text{ Flexural strength} = \frac{231{\cdot}5}{5} = 46{\cdot}3 \text{ in}^3.$$

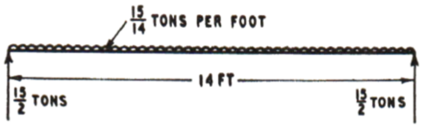
$\therefore$ Ratio of flexural strength I beam to rectangular beam

$$= \frac{46{\cdot}3}{14{\cdot}22} = \underline{3{\cdot}25 : 1}$$

Stresses in a loaded beam are given by

$$\frac{M}{I} = \frac{f}{y}.$$

Maximum bending moment of beam is at the centre. See the sketch.



$$M = \frac{15}{14} \times \frac{14^2}{8} \times \frac{12}{1} \text{ in tons.}$$

$$= 315 \text{ in tons.}$$

but $$f = \frac{My}{I},$$

$$= \frac{315 \times 5}{231{\cdot}5} \text{ tons/in}^2,$$

$$= \underline{6{\cdot}8 \text{ tons/in}^2.}$$

MATHEMATICS C, 1967

Students were expected to answer any six questions.

Q. **1.** *In an experiment the value of y was measured for different x-values as shown below:*

x	25	50	75	150
y	2·18	8·72	19·2	88·6

Assuming a law of the type $y = ax^n$ connecting these two variables, derive a straight-line graph to test this assumption, and hence obtain estimates of the constants a and n.

A. 1. If $y = ax^n$, then

$$\log y = \log a + n \log x,$$

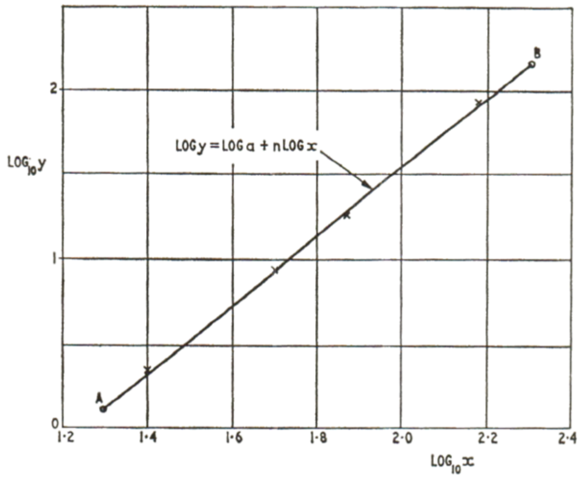
and the logarithmic form of the law is of the linear type $y = mx + c$.

Hence, if the experimental data fit the law $y = ax^n$, the logarithmic form will yield a straight-line graph.

The common logarithms of x and y are tabulated below.

x	25	50	75	150
y	2·18	8·72	19·2	88·6
$\log_{10} x$	1·398	1·699	1·875	2·176
$\log_{10} y$	0·339	0·941	1·283	1·947

The graph of $\log_{10} y$ plotted against $\log_{10} x$ is shown in the sketch, from which it is seen that the plotted points lie substantially on a straight line. Hence, the assumption that the variables x and y are connected by the law $y = ax^n$ is justified.



The gradient of the line is equal to n. Taking the coordinates of points A and B which lie on the line.

$$n = \frac{2\cdot18 - 0\cdot13}{2\cdot3 - 1\cdot3},$$
$$= 2\cdot05.$$

As the graph has a suppressed zero, it is necessary to determine the value of $\log a$ and hence a, by substitution of the known coordinates of a point on the graph and the value of n already determined, into the equation

$$\log y = \log a + n \log x.$$

Taking the coordinates of point A,

$$0\cdot13 = \log a + 2\cdot05 \times 1\cdot3,$$

or,
$$\log a = 0\cdot13 - 2\cdot665,$$
$$= -2\cdot535 \text{ or } \bar{3}\cdot465.$$
$$\therefore \quad a = 0\cdot002917.$$

Thus estimated values of a and n are $\underline{0\cdot00292}$ and $\underline{2\cdot05}$, respectively.

Q. **2.** *Verify, by the use of sketch-graphs or otherwise, that the equation* $x^3 - 10x^2 + 9x + 36 = 0$ *has three distinct real roots.*

By graphical enlargement or otherwise find the largest positive root to three significant figures.

A. 2. $$x^3 - 10x^2 + 9x + 36 = 0.$$

Although the use of a sketch will determine the number and magnitude of the real roots, it is often simpler to try an inspection method, on the assumption that at least one root will be integral. Thus, it is clear that $x = 0$ is not a root because substitution of this value does not satisfy the equation. Simple integral values are then tested as follows:

$x = \pm 1$:
$$x^3 - 10x^2 + 9x + 36 = \pm 1 - 10 \pm 9 + 36.$$

It is seen that neither of these values satisfies the equation

$x = \pm 2$:
$$x^3 - 10x^2 + 9x + 36 = \pm 8 - 40 \pm 18 + 36.$$

Here again $f(x) \neq 0$.

$x = \pm 3$:
$$x^3 - 10x^2 + 9x + 36 = \pm 27 - 90 \pm 27 + 36.$$

Thus when $x = +3$, $f(x) = 0$ and hence $x = 3$ is a root of the equation.

If α, β, and γ denote the roots (real or imaginary) then

$$(x - \alpha)(x - \beta)(x - \gamma) = f(x) = 0.$$
$$\therefore \quad (x - \beta)(x - \gamma) = \frac{f(x)}{x - \alpha}.$$

Hence, the factor $x - 3$ will divide into $x^3 - 10x^2 + 9x + 36$, exactly, to yield a quadratic factor which will contain the other two roots.

$$\begin{array}{r|l}
 & x^2 - 7x - 12 \\
\cline{2-2}
x - 3 & x^3 - 10x^2 + 9x + 36 \\
 & \underline{x^3 - 3x^2} \\
 & - 7x^2 + 9x \\
 & \underline{- 7x^2 + 21x} \\
 & - 12x + 36 \\
 & \underline{- 12x + 36.} \\
 & \underline{- \;- \;-}
\end{array}$$

Hence, $x^3 - 10x^2 + 9x + 36 = (x - 3)(x^2 - 7x - 12) = 0.$

$\therefore \quad x - 3 = 0$ or $x = 3$, already determined,

or, $x^2 - 7x - 12 = 0$, which can be solved as a quadratic by the usual methods.

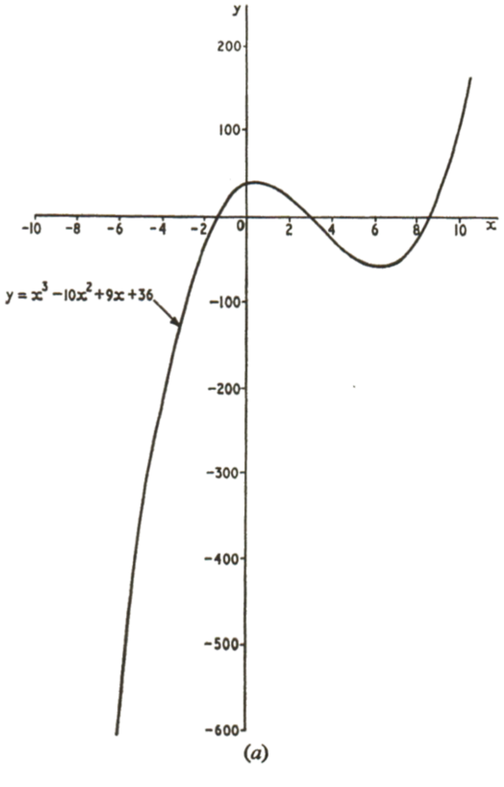
Using the general formula,

$$x = \frac{7 \pm \sqrt{49 + 48}}{2}$$
$$= \frac{7 \pm \sqrt{97}}{2} = \frac{7 \pm 9\cdot849}{2}$$
$$= 8\cdot4245 \text{ or } -1\cdot4245$$

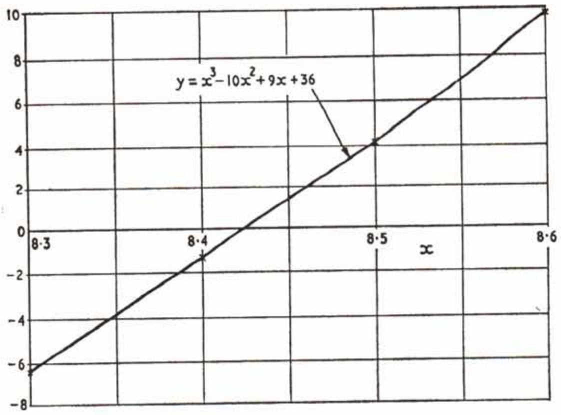
Thus there are three distinct real roots whose values are $-1\cdot4245$, 3 and $8\cdot4245$.

The largest positive root to three significant figures is $\underline{8\cdot42}$.

Note.—This method of inspection is only really feasible when there is at least one real integral root. It avoids the tedious drawing of a sketch graph and graphical enlargement to determine the root more accurately than the sketch would permit. However, in view of the wording of the question and for the sake of completeness, the first (sketch) graph and the enlarged graph in the neighbourhood of $x = 8\cdot4$ are shown as sketches (*a*) and (*b*). The appropriate values



(*a*)



(*b*)

have not been included but the student will no doubt appreciate that the working-out of these, particularly for the enlarged graph, entails quite a lot of effort.

Q. **3.** (*a*) *Obtain as far as the term involving x^4, the binomial series for $(1 + 3x)^{-1/3}$ in ascending powers of x, and give the coefficient of x^r in this series.*

For what range of x is the series valid?

(*b*) *The period t of a simple pendulum (length l) is given by the formula $t = 2\pi\sqrt{l/g}$.*

If the value of the gravitational constant g decreases by k per cent (where k is small) when the pendulum is at a high altitude, show that the corresponding increase in the period is approximately ½k per cent.

A. 3. (*a*)
$$(1 + 3x)^{-1/3} = 1 - \tfrac{1}{3}3x + \frac{-\frac{1}{3}(-\frac{1}{3} - 1)}{1\,.\,2}(3x)^2$$
$$+ \frac{-\frac{1}{3}(-\frac{1}{3} - 1)(-\frac{1}{3} - 2)}{1\,.\,2\,.\,3}(3x)^3$$
$$+ \frac{-\frac{1}{3}(-\frac{1}{3}-1)(-\frac{1}{3}-2)(-\frac{1}{3}-3)}{1\,.\,2\,.\,3\,.\,4}(3x)^4 + \ldots$$
$$= 1 - x + \frac{\frac{1}{3} \times \frac{4}{3}}{1\,.\,2}3^2x^2 - \frac{\frac{1}{3} \times \frac{4}{3} \times \frac{7}{3}}{1\,.\,2\,.\,3}3^3x^3$$
$$+ \frac{\frac{1}{3} \times \frac{4}{3} \times \frac{7}{3} \times \frac{10}{3}}{1\,.\,2\,.\,3\,.\,4}3^4x^4 - \ldots . +$$
$$= \underline{1 - x + 2x^2 - \tfrac{14}{3}x^3 + \tfrac{35}{3}x^4 - \ldots .}$$

Since the divisors, 3, of the numerator of each term cancel out the power of 3 contained in $(3x)^r$, the coefficient of the term containing x^r will be

$$\underline{(-1)^r\,.\,\frac{1\,.\,4\,.\,7\ldots\{1 + 3(r - 1)\}}{1\,.\,2\,.\,3\ldots r}}.$$

The series is valid when $1 > 3x > -1$, i.e., when $\underline{\tfrac{1}{3} > x > -\tfrac{1}{3}}$.

(*b*)
$$t = 2\pi\sqrt{\frac{l}{g}}$$

When the pendulum is at a high altitude, the new value of g will be

$$g - \frac{k}{100}g,$$
$$= g\left(1 - \frac{k}{100}\right).$$

$\therefore$ t', the new value of the period

$$= 2\pi\sqrt{\frac{l}{g\left(1 - \frac{k}{100}\right)}},$$
$$= 2\pi\sqrt{\frac{l}{g}}\left(1 - \frac{k}{100}\right)^{-1/2},$$
$$= t\left\{1 - \tfrac{1}{2} \times \frac{-k}{100} + \ldots\right\}.$$

If k is small,

$$t' = t\left(1 + \frac{k}{200}\right) \text{ ignoring further terms.}$$

Hence t has increased by approximately $\dfrac{k}{2}$ per cent.

Q.E.D.

Q. **4.** (*a*) *Prove*

(i) $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$.
(*ii*) $\cos(x + y)\cos(x - y) = \cos^2 x + \cos^2 y - 1$.

(*b*) *Using the substitution $\tan\theta/2 = t$, express $\sin\theta$ and $\cos\theta$ in terms of t. Hence or otherwise find the values of θ between* 0 *and* 360° *which satisfy the equation $4\cos\theta = 3\sin\theta + 2$.*

A. 4. (*a*) (*i*) $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$.

The left-hand side $= \tan\theta + \cot\theta$,
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta},$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta},$$
$$= \frac{1}{\cos\theta\sin\theta},$$
$$= \sec\theta \operatorname{cosec}\theta. \qquad \text{Q.E.D.}$$

(*ii*) $\cos(x + y)\cos(x - y) = \cos^2 x + \cos^2 y - 1$.

L.H.S. $= \cos(x + y)\cos(x - y)$,
$$= \frac{\cos(x+y+x-y) + \cos\{x+y-(x-y)\}}{2},$$
$$= \frac{\cos 2x + \cos 2y}{2},$$
$$= \frac{2\cos^2 x - 1 + 2\cos^2 y - 1}{2},$$
$$= \cos^2 x + \cos^2 y - 1. \qquad \text{Q.E.D.}$$

(*b*) $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$, from the double-angle formula.

$$= \frac{2\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\cos^2\frac{\theta}{2},$$
$$= \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}},$$
$$= \frac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}, \text{ since } \sec^2\frac{\theta}{2} = 1 + \tan^2\frac{\theta}{2}.$$
$$= \frac{2t}{1 + t^2}, \text{ if, } \tan\frac{\theta}{2} = t.$$

Thus
$$\underline{\sin\theta = \frac{2t}{1 + t^2}}.$$

Also $\cos\theta = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$, from the double-angle formula.

$$= 2\cos^2\frac{\theta}{2} - 1,$$
$$= \frac{2}{1 + \tan^2\frac{\theta}{2}} - 1, \text{ from the first part of } (b),$$
$$= \frac{2}{1 + t^2} - 1,$$
$$= \frac{2 - 1 - t^2}{1 + t^2}.$$

Thus,
$$\underline{\cos\theta = \frac{1 - t^2}{1 + t^2}}.$$

$$4\cos\theta = 3\sin\theta + 2$$

Substituting for $\sin\theta$ and $\cos\theta$ in terms of t from the above expressions,

$$\frac{4(1 - t^2)}{1 + t^2} = \frac{3 \times 2t}{1 + t^2} + 2.$$

or $$4 - 4t^2 = 6t + 2 + 2t^2.$$

$$\therefore \quad 6t^2 + 6t - 2 = 0,$$

or, $$3t^2 + 3t - 1 = 0.$$

Whence, $$t = \frac{-3 \pm \sqrt{9 + 12}}{6},$$
$$= \frac{-3 \pm 4{\cdot}583}{6},$$
$$= 0{\cdot}2638, \text{ or, } -1{\cdot}2638.$$

But, $$t = \tan\frac{\theta}{2}.$$

$$\therefore \frac{\theta}{2} = 14°47' \text{ or } 194°47', \text{ when } t = 0{\cdot}2638.$$

or $$128°21' \text{ or } 308°21', \text{ when } t = -1{\cdot}2638.$$

Thus, $\theta = \underline{29°34' \text{ or } 256°42'}$ for values between 0° and 360°.

Q. 5. *The formula*

$$y = \frac{c}{2}(e^{x/2c} + e^{-x/2c}) - c$$

gives the sag y ft in the centre of a length of suspended wire, where x ft is the distance between the points of suspension, and c is a constant.

If c = 450, calculate the sag in a 90-ft span of the wire.

If x is very much less than c, show that y is very nearly proportional to x^2.

A. 5. $$y = \frac{c}{2}(\mathrm{e}^{x/2c} + \mathrm{e}^{-x/2c}) - c.$$

When $$c = 450 \text{ and } x = 90,$$
$$y = \frac{450}{2}(\mathrm{e}^{90/900} + \mathrm{e}^{-90/900}) - 450,$$
$$= 225(\mathrm{e}^{0\cdot1} + \mathrm{e}^{-0\cdot1}) - 450.$$

Using a table of exponential functions, this may be calculated directly as
$$y = 225(1{\cdot}1052 + 0{\cdot}9048) - 450,$$
$$= 225 \times 2{\cdot}01 - 450.$$

$\therefore$ The sag $= \underline{2{\cdot}25 \text{ ft.}}$

If such a table is not available, common logarithms must be used as follows:
$$\log_{10}\mathrm{e}^{\pm0\cdot1} = \pm0{\cdot}1 \times 0{\cdot}4343 = \pm0{\cdot}0434.$$
$$\therefore\ \mathrm{e}^{0\cdot1} = 1{\cdot}105 \text{ and } \mathrm{e}^{-0\cdot1} = 0{\cdot}9048.$$
$$\therefore\ y = 225\,(1{\cdot}105 + 0{\cdot}9048) - 450, \text{ as previously.}$$

The series for e^x is given as
$$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.$$

Hence $$\mathrm{e}^{x/2c} = 1 + \frac{x}{2c} + \left(\frac{x}{2c}\right)^2\frac{1}{2!} + \ldots,$$
$$= 1 + \frac{x}{2c} + \frac{x^2}{8c^2} + \ldots,$$

and $$\mathrm{e}^{-x/2c} = 1 - \frac{x}{2c} + \left(\frac{x}{2c}\right)^2\frac{1}{2!} - \ldots,$$
$$= 1 - \frac{x}{2c} + \frac{x^2}{8c^2} - \ldots.$$

If $x \ll c$, then, $\mathrm{e}^{x/2c} \simeq 1 + \frac{x}{2c} + \frac{x^2}{8c^2}$ neglecting terms in x^3 and higher powers of x

Similarly,
$$\mathrm{e}^{-x/2c} \simeq 1 - \frac{x}{2c} + \frac{x^2}{8c^2}.$$

Hence, $$y \simeq \frac{c}{2}\left(1 + \frac{x}{2c} + \frac{x^2}{8c^2} + 1 - \frac{x}{2c} + \frac{x^2}{8c^2}\right) - c,$$
$$= \frac{c}{2}\left(2 + \frac{x^2}{4c^2}\right) - c,$$
$$= c + \frac{x^2}{8c} - c,$$
$$= \frac{x^2}{8c}.$$

Thus $y \propto x^2$, c being constant. Q.E.D.

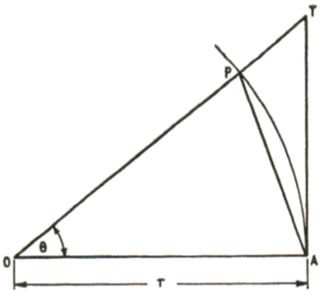
Q. 6. (*a*) *Show that if θ is acute and measured in radians, then sin θ/θ lies between the values cos θ and* 1.

(*b*) *Prove from first principles that*
$$\frac{d}{dx}(\mathit{cosec}\ x) = -\ \mathit{cosec}\ x \cot x.$$

(*c*) *If* $y = x^{-1/2} \sin x$, *prove that*
$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \tfrac{1}{4})y = 0.$$

A. 6. (*a*) In the sketch let ∠ POA be an acute angle of θ radians subtended by the arc AP of a circle of centre 0 and radius *r*.

Let the tangent to the circle at A meet OP produced at T. Join AP.



Then

Area of triangle AOP $= \frac{1}{2}$OA OP $\sin\theta = \frac{1}{2}r^2\sin\theta$.

Area of sector AOP $= \frac{1}{2}r^2\theta$.

Area of triangle AOT $= \frac{1}{2}$OA AT $= \frac{1}{2}r^2\tan\theta$.

Now, area of triangle AOP < area of sector AOP < area of triangle AOT.

$$\therefore\ \tfrac{1}{2}r^2\sin\theta < \tfrac{1}{2}r^2\theta < \tfrac{1}{2}r^2\tan\theta,$$

or, $$\sin\theta < \theta < \tan\theta.$$

Dividing by $\sin\theta$,
$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta},$$

whence, $$1 > \frac{\sin\theta}{\theta} > \cos\theta.$$

Thus, provided θ is acute, $\frac{\sin\theta}{\theta}$ must lie between $\cos\theta$ and 1.

(*b*) Let $y = \operatorname{cosec} x$.

Suppose x increases by a small amount δx and let δy be the corresponding change in y.

$$\therefore\ y + \delta y = \operatorname{cosec}(x + \delta x),$$
$$= \frac{1}{\sin(x + \delta x)}$$
$$\therefore\ \delta y = \frac{1}{\sin(x + \delta x)} - \frac{1}{\sin x}, \text{ since } y = \frac{1}{\sin x},$$
$$= \frac{\sin x - \sin(x + \delta x)}{\sin x \sin(x + \delta x)},$$
$$= -\frac{\sin(x + \delta x) - \sin x}{\sin x \sin(x + \delta x)},$$
$$= -\frac{2\cos\frac{2x + \delta x}{2}\sin\frac{\delta x}{2}}{\sin x \sin(x + \delta x)}.$$
$$\therefore\ \frac{\delta y}{\delta x} = -\frac{\cos\left(x + \frac{\delta x}{2}\right)\dfrac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}}{\sin x \sin(x + \delta x)}.$$

But $$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{\delta x \to 0}\frac{\delta y}{\delta x}.$$

$$\therefore\ \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\cos x}{\sin^2 x} \quad \text{since } \lim_{\delta x \to 0}\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}} = 1.$$
$$= -\operatorname{cosec} x \cot x.$$
Q.E.D.

(*c*)
$$\begin{aligned}
y &= x^{-1/2}\sin x.\\
\therefore\ \frac{dy}{dx} &= x^{-1/2}\cos x - \tfrac{1}{2}x^{-3/2}\sin x,\\
\text{and,}\quad \frac{d^2y}{dx^2} &= x^{-1/2}(-\sin x) - \tfrac{1}{2}x^{-3/2}\cos x\\
&\quad - \tfrac{1}{2}\{x^{-3/2}\cos x - \tfrac{3}{2}x^{-5/2}\sin x\}.\\
&= -x^{-1/2}\sin x - \tfrac{1}{2}x^{-3/2}\cos x - \tfrac{1}{2}x^{-3/2}\cos x\\
&\quad + \tfrac{3}{4}x^{-5/2}\sin x,\\
&= -x^{-1/2}\sin x - x^{-3/2}\cos x + \tfrac{3}{4}x^{-5/2}\sin x.
\end{aligned}$$

Hence,
$$\begin{aligned}
&x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \tfrac{1}{4})y,\\
&= x^2(-x^{-1/2}\sin x - x^{-3/2}\cos x + \tfrac{3}{4}x^{-5/2}\sin x)\\
&\quad + x(x^{-1/2}\cos x - \tfrac{1}{2}x^{-3/2}\sin x)\\
&\quad + (x^2 - \tfrac{1}{4})x^{-1/2}\sin x,\\
&= -x^{3/2}\sin x - x^{1/2}\cos x + \tfrac{3}{4}x^{-1/2}\sin x\\
&\quad + x^{1/2}\cos x - \tfrac{1}{2}x^{-1/2}\sin x\\
&\quad + x^{3/2}\sin x - \tfrac{1}{4}x^{-1/2}\sin x,\\
&= \tfrac{1}{2}x^{-1/2}\sin x - \tfrac{1}{2}x^{-1/2}\sin x\\
&= 0. \qquad\qquad \text{Q.E.D.}
\end{aligned}$$

***Q.* 7.** *(a) Differentiate with respect to x*

(i) $\dfrac{2x-3}{(x+2)^3}$ *(ii)* $(x^2 - 5x - 4)e^{-3x}$.

Simplify the result in each case.

(b) The demand for new television sets in a town is assumed to be proportional directly to the number of families owning a set, and to the cube of the number not yet owning one.

Assuming a fixed population, find the proportion of families owning a set when the demand is greatest.

A. 7. (*a*) (*i*) Let $y = \dfrac{2x-3}{(x+2)^3}$.

$$\begin{aligned}
\text{Then}\quad \frac{dy}{dx} &= \frac{(x+2)^3 2 - (2x-3)3(x+2)^2}{(x+2)^6},\\
&= \frac{2(x+2) - 3(2x-3)}{(x+2)^4},\\
&= \frac{4x + 4 - 6x + 9}{(x+2)^4},\\
&= \underline{\frac{13-2x}{(x+2)^4}}
\end{aligned}$$

(*ii*) Let $y = (x^2 - 5x - 4)e^{-3x}$.

$$\begin{aligned}
\text{Then}\quad \frac{dy}{dx} &= (x^2 - 5x - 4)(-3)e^{-3x}\\
&\quad + (2x - 5)e^{-3x},\\
&= e^{-3x}(-3x^2 + 15x + 12 + 2x - 5),\\
&= \underline{e^{-3x}(-3x^2 + 17x + 7)}
\end{aligned}$$

(*b*) Let p be the fixed population of families and let S be the number of families owning television sets. Hence $p - S$ will be the number of families not yet owning sets.

If D represents the demand, then

$$\begin{aligned}
&D \propto S,\\
\text{and, } &D \propto (p - S)^3.\\
\therefore\ &D = kS(p - S)^3, \text{ where, } k \text{ is a constant.}
\end{aligned}$$

Since D, S, and p can all be assumed positive numbers and since $p > S$, it follows that k must also be positive.

$$\begin{aligned}
\text{Now}\quad \frac{dD}{dS} &= kS3(p - S)^2(-1) + k(p - S)^3,\\
&= -3kS(p - S)^2 + k(p - S)^3,\\
&= (p - S)^2(-3kS + kp - kS),\\
&= k(p - S)^2(p - 4S).
\end{aligned}$$

For maximum or minimum values of $D, \dfrac{dD}{dS} = 0$.

$\therefore\ k(p - S)^2(p - 4S) = 0,$

whence, $p - S = 0$,

which means that all families own television sets and hence the demand is a minimum.

or, $p - 4S = 0$, i.e. $S = \dfrac{p}{4}$.

In order to check whether this is a maximum or a minimum, it is perhaps simplest to find the second derivative.

$$\begin{aligned}
\text{Thus}\quad \frac{d^2D}{d_2S} &= k(p - S)^2(-4) + 2k(p - S)(-1)(p - 4S),\\
&= (p - S)[-4k(p - S) - 2k(p - 4S)],\\
&= (p - S)(-6kp + 12kS),
\end{aligned}$$

When $S = \dfrac{p}{4}$,

$$\begin{aligned}
\frac{d^2D}{d_2S} &= \frac{3p}{4}(-6k)\left(p - \frac{p}{2}\right),\\
&= \frac{-9k}{2}\,\frac{p^2}{2} = \frac{-9kp^2}{4}.
\end{aligned}$$

As k and p^2 are positive, $\dfrac{d^2D}{d_2S}$ is negative and hence D is a maximum when $S = \dfrac{p}{4}$.

Thus the proportion of families owning a set is one quarter when the demand is greatest.

***Q.* 8.** (*a*) *Evaluate the integrals*

(*i*) $\displaystyle\int_2^3 \frac{dx}{2x-1}$ and (*ii*) $\displaystyle\int_0^{\pi/3} \sin 4x\cos 2x dx$.

(b) P, Q, R are the three points (− h, p), (O, q) and (h, r) in the x, y plane. A parabola of the form $y = ax^2 + bx + c$ *is drawn through P, Q, and R. Prove that the area between the parabola, the x-axis and the ordinates x = − h and x = + h is given by* $\dfrac{h}{3}(p + 4q + r)$.

A. 8. (*a*) (*i*)
$$\begin{aligned}
\int_2^3 \frac{dx}{2x-1} &= \left[\tfrac{1}{2}\log_e(2x-1)\right]_2^3,\\
&= \tfrac{1}{2}(\log_e 5 - \log_e 3),\\
&= \tfrac{1}{2}(1{\cdot}6094 - 1{\cdot}0986).\\
&= \tfrac{1}{2}\times 0{\cdot}5108 = \underline{0{\cdot}2554.}
\end{aligned}$$

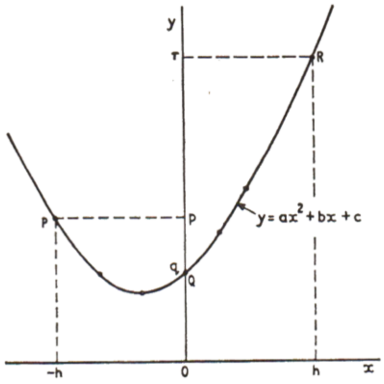
(*ii*) $\displaystyle\int_0^{\pi/3} \sin 4x\cos 2x dx.$

Using the product-to-sum trigonometrical formula

$$\begin{aligned}
2\sin A\cos B &= \sin(A + B) + \sin(A - B),\\
\sin 4x\cos 2x &= \tfrac{1}{2}\{\sin 6x + \sin 2x\},
\end{aligned}$$

Hence,
$$\begin{aligned}
&\int_0^{\pi/3} \sin 4x\cos 2x dx\\
&= \tfrac{1}{2}\int_0^{\pi/3}(\sin 6x + \sin 2x)dx,\\
&= \tfrac{1}{2}\left[\frac{-\cos 6x}{6} - \frac{\cos 2x}{2}\right]_0^{\pi/3},\\
&= \tfrac{1}{2}\left[\frac{-\cos 2\pi}{6} - \frac{\cos\frac{2\pi}{3}}{2} + \frac{\cos 0}{6} + \frac{\cos 0}{2}\right],\\
&= \tfrac{1}{2}[-\tfrac{1}{6} + \tfrac{1}{4} + \tfrac{1}{6} + \tfrac{1}{2}],\\
&= \underline{3/8.}
\end{aligned}$$

(*b*) A typical parabola of the form $y = ax^2 + bx + c$, is shown in the sketch, passing through the points P, Q, and R. Since these three points are on the curve, their coordinates must satisfy the equation $y = ax^2 + bx + c$.



$$\therefore \quad p = ah^2 - bh + c \ldots \text{point P.}$$
$$q = c \ldots\ldots\ldots\ldots \text{point Q.}$$
$$r = ah^2 + bh + c \ldots \text{point R.}$$

Substituting q for c in the first and third equations:

$$p = ah^2 - bh + q,$$
$$r = ah^2 + bh + q.$$

Subtracting p from r.

$$r - p = 2bh,$$

or, $$b = \frac{r-p}{2h}.$$

Substituting for b and c in the equation for point R,

$$r = ah^2 + \frac{r-p}{2h}h + q,$$

or, $$2r = 2ah^2 + r - p + 2q.$$

$$\therefore \quad 2ah^2 = p + r - 2q,$$

or, $$a = \frac{p + r - 2q}{2h^2}.$$

Thus the equation for the parabola becomes,

$$y = \frac{p+r-2q}{2h^2}x^2 + \frac{r-p}{2h}x + q \text{ on substituting}$$

for a, b, and c.

The area between the parabola, the x-axis and the ordinates $x = -h$ and $x = h$ is given by,

$$\begin{aligned}
\int_{-h}^{h} y\,dx &= \int_{-h}^{h}\left(\frac{p+r-2q}{2h^2}x^2 + \frac{r-p}{2h}x + q\right)\mathrm{d}x,\\
&= \left[\frac{p+r-2q}{2h^2}\frac{x^3}{3} + \frac{r-p}{2h}\frac{x^2}{2} + qx\right]_{-h}^{h},\\
&= \frac{p+r-2q}{2h^2}\left(\frac{h^3}{3} + \frac{h^3}{3}\right) + \frac{r-p}{2h}\left(\frac{h^2}{2} - \frac{h^2}{2}\right)\\
&\qquad + qh + qh,\\
&= \frac{h}{3}(p + r - 2q) + 2qh,\\
&= \frac{h}{3}(p + r - 2q + 6q),\\
&= \frac{h}{3}(p + 4q + r). \qquad \text{Q.E.D.}
\end{aligned}$$

Q. **9.** (*a*) *Sketch the graph* $y = 5 \cos 3\omega t$ *from* $t = 0$ *to* $t = \frac{2\pi}{\omega}$, *and obtain*

(*i*) *the mean value,*

(*ii*) *the root-mean-square value of* y *from* $t = 0$ *to* $t = \frac{\pi}{2\omega}$.

(*b*) *Find the volume generated by the complete revolution about the x-axis of the curve* $y = e^{-kx}$, *between* $x = 0$ *and* $x = \frac{1}{k}$.

A. 9. (*a*) (*i*) The mean value of the expression $= \underline{3{\cdot}183.}$

(*ii*) The root-mean-square value of y from

$$t = 0 \text{ to } t = \frac{\pi}{2\omega} = \underline{3{\cdot}536.}$$

(*b*) The required volume

$$= \underline{\frac{1{\cdot}358}{k}}.$$

Q. **10.** (*a*) *If* $z = r(\cos\theta + j\sin\theta) = r\angle\underline{\theta}$ *prove that*

$$z_1z_2 = r_1r_2\angle\underline{\theta_1 + \theta_2}.$$

(*b*) *Express* $\frac{1 - \frac{1}{3}j}{1 + j}$ *in both polar and cartesian forms as a single complex number.*

(*c*) *Obtain the two square roots of the complex number* $-4 + 3j$ *in the form* $a + jb$.

A. 10. (*a*) $$z = r(\cos\theta + \mathrm{j}\sin\theta) = r\angle\underline{\theta}.$$

If $z_1 = r_1\angle\underline{\theta_1}$ and $z_2 = r_2\angle\underline{\theta_2}$,

then, $$\begin{aligned}
z_1z_2 &= r_1(\cos\theta_1 + \mathrm{j}\sin\theta_1) \times r_2(\cos\theta_2 + \mathrm{j}\sin\theta_2),\\
&= r_1r_2\{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\\
&\qquad + \mathrm{j}\sin\theta_1\cos\theta_2 + \mathrm{j}\sin\theta_2\cos\theta_1\},\\
&= r_1r_2\{\cos(\theta_1 + \theta_2) + \mathrm{j}\sin(\theta_1 + \theta_2)\}.
\end{aligned}$$

$$\therefore \; z_1z_2 = r_1r_2\angle\underline{\theta_1 + \theta_2} \qquad \text{Q.E.D.}$$

(*b*) $$\begin{aligned}
\frac{1 - \frac{1}{3}\mathrm{j}}{1 + \mathrm{j}} &= \frac{(1 - \frac{1}{3}\mathrm{j})(1 - \mathrm{j})}{(1 + \mathrm{j})(1 - \mathrm{j})},\\
&= \frac{1 - \frac{1}{3}\mathrm{j} - \mathrm{j} - \frac{1}{3}}{1 + 1},\\
&= \frac{\frac{2}{3} - \frac{4}{3}\mathrm{j}}{2},\\
&= \underline{\tfrac{1}{3} - \tfrac{2}{3}\mathrm{j}}, \text{ in Cartesian form}\\
&= \sqrt{(\tfrac{1}{3})^2 + (\tfrac{2}{3})^2}\;\angle\underline{\tan^{-1} - \frac{2/3}{1/3}},\\
&= \frac{\sqrt{5}}{3}\angle\underline{\tan^{-1} - 2},\\
&= \underline{0{\cdot}745\,\angle -63°26'} \text{ in polar form.}
\end{aligned}$$

(*c*) $-4 + 3\mathrm{j} = r(\cos\theta + \mathrm{j}\sin\theta)$,

where $$r = \sqrt{4^2 + 3^2} = 5,$$

and $$\theta = \tan^{-1}\left(\frac{3}{-4}\right) = 143°8', \text{ since } \theta \text{ is in the second quadrant.}$$

$$\therefore \quad -4 + 3\mathrm{j} = 5\{\cos(143°8' + n\,360°) + \mathrm{j}\sin(143°8' + n\,360°)\},$$

where, $$n = 0, 1, 2 \ldots.$$

Hence,

$$\begin{aligned}
(-4 + 3\mathrm{j})^{1/2} &= \sqrt{5}\left\{\cos\frac{(143°8' + n\,360°)}{2} + \mathrm{j}\sin\frac{(143°8' + n\,360°)}{2}\right\},\\
&= \sqrt{5}\{\cos(71°34' + n\,180°) + \mathrm{j}\sin(71°34' + n\,180°)\}.
\end{aligned}$$

Giving n the values 0 and 1, the two square roots are

$$\sqrt{5}(\cos 71°34' + \mathrm{j}\sin 71°34') \text{ and } \sqrt{5}(\cos 251°34' + \mathrm{j}\sin 251°34')$$

or, $\sqrt{5}(0{\cdot}3162 + \mathrm{j}\,0{\cdot}9487)$ and $\sqrt{5}(-0{\cdot}3162 - \mathrm{j}\,0{\cdot}9487)$.

Hence $$(-4 + 3\mathrm{j})^{1/2} = \underline{0{\cdot}7071 + \mathrm{j}\,2{\cdot}121} \text{ and } \underline{-0{\cdot}7071 - \mathrm{j}2{\cdot}121.}$$

Note.—It is incorrect to use the polar form in determining the square root, i.e. to say that $\sqrt{r\angle\underline{\theta}} = \pm\sqrt{r}\angle\underline{\theta/2}$. The modulus is intrinsically positive and the angles of the roots cannot readily be determined from the polar form. The student must always use the trigonometric form as above.

LINE TRANSMISSION C, 1967

Students were expected to answer any six questions

Q. **1.** *Explain what is meant by the characteristic impedance of a uniform transmission line and describe how it can be determined by bridge measurements at one end of the line.*

A uniform line has the following primary coefficients at $\omega = 5{,}000$ *rad/s:* $L = 1$ *mH/mile,* $R = 40$ *ohms/mile,* $C = 0{\cdot}1$ *μF/mile,* $G = 1$ *μmho/mile. Calculate its characteristic impedance at this frequency.*

A. 1. The characteristic impedance of a uniform transmission line is the impedance which would be measured at the input if the line were infinitely long. It follows that if a finite length of line were to be terminated in its characteristic impedance the line would appear to be of infinite length.

For a uniform line the characteristic impedance Z_0 is given by the expression

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ ohms,}$$

at an angular frequency of ω radians/second.

R, L, G and C are the primary coefficients: R is the resistance in ohms, L is the inductance in henrys, G is the leakance in mhos and C is the capacitance in farads per loop mile.

The characteristic impedance can be determined by measuring the input impedance of the line with the distant end (*a*) open circuit, and (*b*) short circuit, and taking the geometric mean of the two measurements.

i.e. $\underline{Z_0 = \sqrt{Z_{oc} \times Z_{sc}}}$.

For the values given:

$$R + j\omega L = 40 + j5 = 40\cdot 31\,\underline{|7\cdot 1^\circ},$$

$$G + j\omega C = 10^{-6}(1 + j500) = 500 \times 10^{-6}\,\underline{|89\cdot 9^\circ}.$$

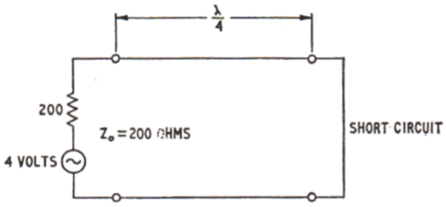
$$\therefore \quad |Z_\theta| = \sqrt{\frac{40\cdot 31}{500 \times 10^{-6}}} = 10^3\sqrt{\frac{40\cdot 31}{500}} = 284 \text{ ohms}$$

and, $$\phi = \frac{7\cdot 1^\circ - 89\cdot 9^\circ}{2} = -41\cdot 4^\circ.$$

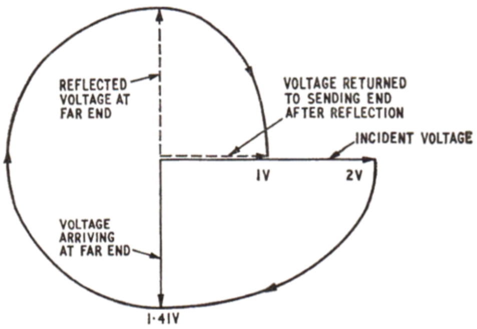
Hence, $\underline{Z_0 = 284\,\overline{|41\cdot 4^\circ} \text{ ohms.}}$

Q. 2. *At a particular frequency a cable pair has a characteristic impedance of 200 ohms and is one-quarter of a wavelength long. When terminated in its characteristic impedance it has a transmission loss of 3 dB. Calculate its impedance as seen from the sending end when the far end is short-circuited.*

A. 2. Assume the line to be fed by a generator having an internal impedance of 200 ohms and an e.m.f. of 4 volts as shown in sketch (*a*). The incident voltage will be 2 volts and the incident current 10 mA.



(*a*)


REFLECTED VOLTAGE AT FAR END
VOLTAGE RETURNED TO SENDING END AFTER REFLECTION
INCIDENT VOLTAGE
1V
2V
VOLTAGE ARRIVING AT FAR END
1·41V

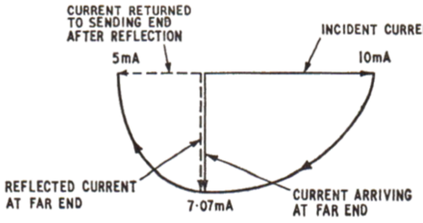
(*b*)

Sketch (*b*) shows the phasor (vector) diagram for voltage. On reaching the far end, the voltage wave will have changed in phase by $\frac{\pi}{2}$ radians (90°) and it will have been attenuated by 3 dB. The wave will be reversed in phase (180°) at the short-circuited end and it will arrive back at the sending end after a further change in phase of $\frac{\pi}{2}$ radians and a further attenuation of 3 dB. The voltage wave returning to the sending end will thus have been attenuated by 6 dB (becoming 1 volt) but it will be in phase again with the incident voltage. The total voltage at the sending end will thus be 2 + 1 = 3 volts.

Sketch (*c*) shows the phasor (vector) diagram for current. On reaching the far end the current wave will have been attenuated by 3 dB and it will have changed in phase by 90°. No change in phase will occur at the short-circuited end and it will return to the sending end after a further change in phase of 90° and a further attenuation of 3 dB. The returned current will thus be 5 mA lagging 180° on the incident current. Hence the total current at the sending end will be 10 − 5 = 5 mA.

$$\text{Sending end impedance} = \frac{\text{Total voltage}}{\text{Total current}} \text{ at sending end}$$

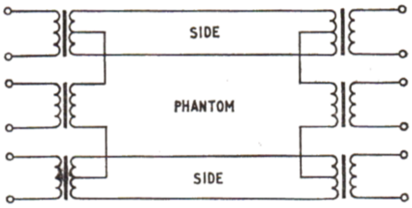
$$= \frac{3 \text{ volts}}{5 \text{ mA}} = \underline{600 \text{ ohms.}}$$


CURRENT RETURNED TO SENDING END AFTER REFLECTION
5mA
INCIDENT CURRENT
10mA
REFLECTED CURRENT AT FAR END
7·07mA
CURRENT ARRIVING AT FAR END

(*c*)

Q. 3. *Show how superposed (phantom) channels can be provided over pairs of a trunk or junction cable. Explain how side circuits and phantom circuits can be loaded independently.*

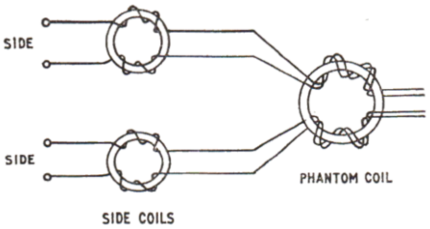
A. 3. Sketch (*a*) shows two pairs over which a phantom circuit is provided by using the centre-tap on the line side of each line transformer. Each leg of the phantom circuit consists of the two wires of a pair in parallel. With such an arrangement each pair is termed a "side" circuit.



(*a*)

No interference will occur between the phantom and side circuits if the phantom-circuit current divides equally between the two wires of each side circuit. Thus it is essential that the centre tapping should be accurate and the two wires of each pair equal in impedance. The phantom circuit requires its own line transformers for impedance matching unless the phantom is to be used only for d.c. signalling.

Phantom circuits for speech or music may be provided over the two pairs of a multiple-twin cable (now obsolescent in the British Post Office) or over carrier quad-cable. Those over audio quad-cable are only suitable for d.c. signalling.



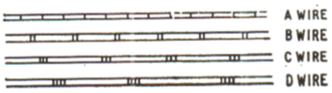
(*b*)

Sketch (*b*) shows how a phantom circuit may be loaded using a specially-wound coil. This has a bifilar winding and is wound in such a way that it only presents inductance to current in the phantom circuit; it is non-inductive as far as the side circuits are concerned. It is usual to put the side-circuit coils and the phantom-circuit coils in the same loading pot and mutual interference by inductive coupling is prevented by using toroidal cores giving a negligible external magnetic-field.

Q. 4. *What steps are taken to minimize crosstalk in (a) the manufacture and (b) the installation of a quad trunk cable intended for carrier working?*

Explain the difference between near-end crosstalk and far-end crosstalk and show how they can be reduced to an acceptably low level in a cable of this type.

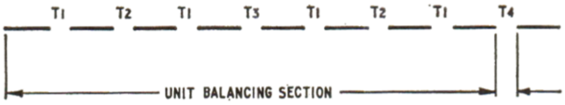
A. 4. *Manufacture.* Great care is taken during manufacture to achieve uniformity. The four wires of a quad are taken from the same reel of wire as supplied thus ensuring that all the wire for one quad is drawn through the same die. There might be a slight difference in diameter between dies due to wear. The insulating paper is supplied in large rolls and each is printed with identification stripes before being slit into narrow ribbons suitable for covering the wire. The identification stripes are printed diagonally so that when the paper is lapped helically over the wire the identification appears in the form of one, two, three or four rings as shown in sketch (*a*). The spacing between the stripes is arranged so that the same amount of ink is used for each wire of a quad over a given length; this obviates any difference in insulation resistance which might be caused by the presence of the ink.



(*a*)

For trunk cables it is usual to use paper ribbon cut from the same large roll for each wire of a quad and to insulate each wire in succession on the same machine. The four wires of a quad are then laid up as a quad ready for incorporation in the cable. For a carrier cable all quads have different lengths of lay, some being laid clockwise and some anti-clockwise. The cable is built up in layers of quads, each layer being laid helically over the one below it and in the opposite direction. All quads in any one layer have the same direction of quadding-lay but adjacent layers have clockwise and anti-clockwise quads.

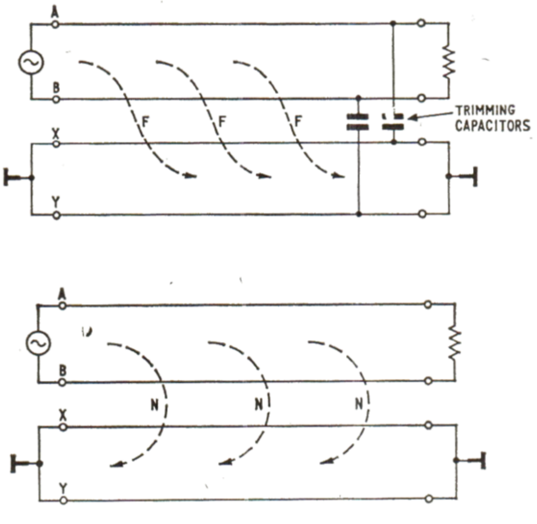
Installation. Every joint in a carrier cable is test selected on the basis of capacitance unbalance measurements on each 176 yard length.



(*b*)

Sketch (*b*) shows a typical unit balancing section comprising eight lengths. These are first jointed in groups of two (joints T1), then in groups of four (joints T2). Joint T3 between the two groups of four completes the unit balancing section. A number of such sections are then selectively jointed (joints T4) to build up the cable to the required length.

Crosstalk. Sketch (*c*) shows how crosstalk occurs between channels. Part of the signal applied to pair AB appears on pair XY as near-end and far-end crosstalk. All the various paths for far-end crosstalk have the same electrical length and thus the entire crosstalk signal bears the same phase relationship with the original signal and it can be reduced to an acceptably low level by the addition of small trimming capacitors at the far end. For near-end crosstalk such a solution is not possible because all the paths have different electrical lengths and thus the components of the crosstalk signal cover a range of phase relationships. For this reason it is usual to have a separate cable for each direction of transmission so that the lead sheaths form an effective screen and provide a high attenuation in the near-end crosstalk path.



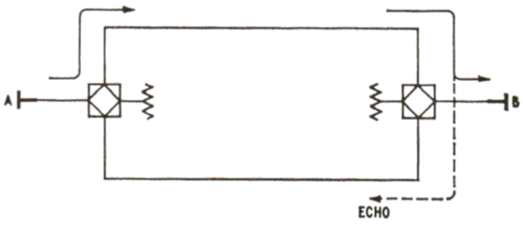
(*c*)

Q. **5.** *Draw a block schematic diagram of a 4-wire repeater circuit between two terminal repeater stations and passing through one intermediate repeater station. Sketch a typical transmission level diagram for one direction of transmission at frequencies of 300 Hz and 3,400 Hz assuming the overall loss between 2-wire terminations to be 3 dB. What factors determine the maximum and minimum levels?*

A. 5. See A. 7, Line Transmission C, 1965, *Supplement*, Vol. 59, No. 3, p. 50, Oct. 1966.

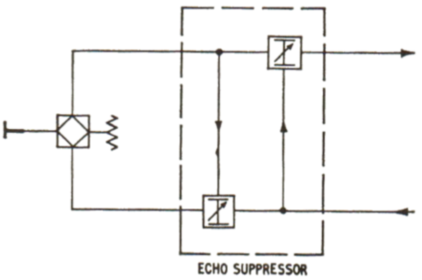
Q. **6.** *Describe the circumstances which require an echo suppressor to be fitted to a telephone circuit. Give the basic circuit diagram of an echo suppressor and explain its operation.*

A. 6. Where 4-wire circuits are used it is necessary to provide 2-wire/4-wire terminating sets each comprising a hybrid transformer and line balancing network. Sketch (*a*) shows a 4-wire connexion between two subscribers A and B. Assume subscriber A to be speaking over the path shown by the full arrows. Owing to the inevitable imperfection of the line-balancing networks a small part of the signal received at the B-end termination passes across the hybrid transformer and appears at the input of the B-to-A channel. It is thus transmitted back to subscriber A's receiver.



(*a*)

Where the connexion is fairly short the round-trip delay time is small and subscriber A merely hears the returned signal as sidetone. Where the connexion is a long one, however, and the round-trip delay time is appreciable (say 200 ms), the signal appears as a distinct echo of his own voice. Of course, subscriber B hears a similar echo when he himself is talking and under these conditions conversation is very difficult. On such circuits echo suppressors are fitted in order to attenuate the return path when a signal is present on the forward path. In this way the echo is reduced to an acceptable level. Sketch (*b*) shows the arrangement.

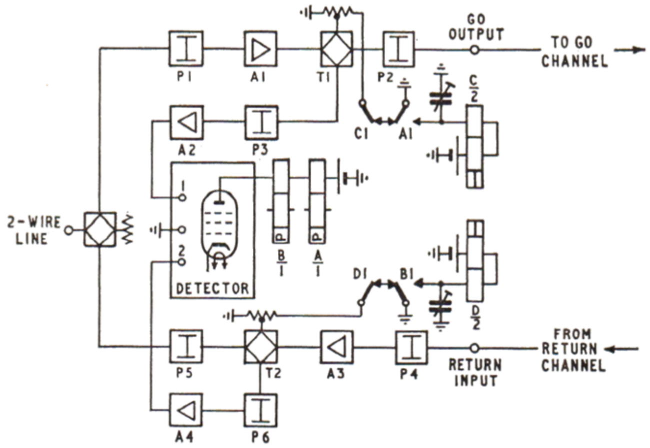


(*b*)

Echo suppressors were originally used on long 4-wire audio circuits, but these have now been largely replaced by high-velocity carrier circuits where the round-trip delay time is small. At the present time echo suppressors are commonly used on ocean cable or satellite circuits.

Sketch (*c*) is a block-schematic diagram of an echo suppressor used on transoceanic circuits. T1 and T2 are terminating sets in the "go" and "return" transmission paths and their insertion loss can be changed by short-circuiting their balances. In the idle condition (as shown) both balances are short-circuited and thus the insertion loss is small. The presence of a speech signal in either the "go" or the "return" channel energizes the detector via amplifier A2 or amplifier A4. The detector is designed so that a signal reaching terminal 1 causes relay B to operate while a signal reaching terminal 2 causes relay A to operate. Thus if a speech signal is present on the "go" channel part of it will reach terminal 1 of the detector and relay B will operate. This will remove the short circuit from the balance of terminating set T2 thus making its balance effective and increasing the insertion loss substantially. Similarly, speech in the "return" channel causes relay A to operate to make the balance of T1 effective.

Relays C and D give some delay in restoring the terminating sets to their normal condition. For example, on the operation of relay B its contact completes the circuit for relay D. On the release of B, when the speech signal ceases, relay D remains operated while its capacitor is recharged and contact D delays the replacement of the short-circuit on the balance of terminating set T2. This gives a hang-over time (the time taken to restore conditions to normal) which can be adjusted within the range 15 to 100 ms.



(*c*)

Q. **7.** *What are the basic requirements of a group amplifier suitable for use on a* 12-*channel C.C.I.T.T. group? Sketch a typical circuit arrangement, and show how these requirements are met.*

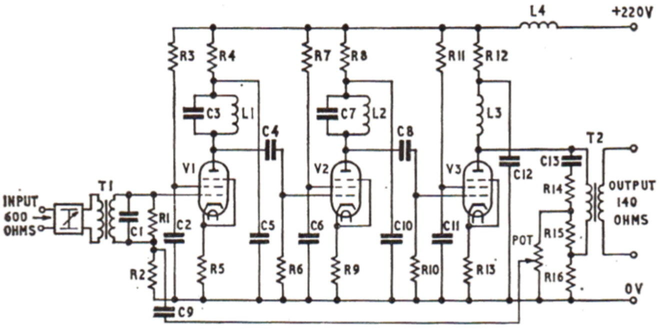
A. 7. The basic requirements of a group amplifier are:

(*a*) To amplify all channels without introducing intermodulation between any of them,

(*b*) To give a flat frequency response so that all channels are amplified equally,

(*c*) To be stable in operation.

There are two basic designs, one using thermionic valves and the other using transistors. The sketch shows a simplified version of the valve amplifier. This is a 3-stage amplifier having a maximum gain of 65 dB which is adjustable in ½ dB steps. Each stage has local current feedback and composite feedback is provided overall. The input impedance is 600 ohms and the output impedance is 140 ohms.



Requirement (*a*) is met by designing the circuits in such a way that no valves or transformers are overloaded; (*b*) is met by proper adjustment of the tuned circuits (the input transformer and the anode loads) and (*c*) is met by the use of negative feedback.

Q. **8.** *Define balance return loss and explain how it can be measured. Calculate the balance return loss between a line having a characteristic impedance of* 800 $\angle -30°$ *ohms and a network having an impedance of* (600 − j50) *ohms.*

A. 8. Balance-return loss is a measure of the accuracy with which a balancing network is constructed to match the impedance of the line it is intended to simulate.

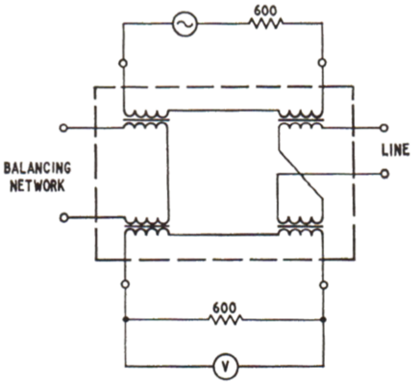
For a balancing network having an impedance Z_B and a line having an impedance Z_0,

$$\text{balance return loss} = 20 \log \left|\frac{Z_o + Z_B}{Z_o - Z_B}\right| \text{dB.}$$

When $Z_o = Z_B$ the balance return loss is infinite and no reflexion occurs.

The sketch shows how the balance return loss may be measured using the hybrid transformer with which it is to be associated in a 2-wire/4-wire terminating set. Two voltage measurements are made—the first with the balancing network not connected (reading V_1) and the second with the balance connected (reading V_2).

$$\text{Balance return loss} = 20 \log \frac{V_1}{V_2} \text{dB.}$$



If a level measuring set is used, the result is given as the difference between the two readings in dB.

For the values given

$$\begin{aligned} Z_0 &= 800 \cos(-30°) + \text{j}800 \sin(-30°) \\ &= 693 - \text{j}400 \end{aligned}$$

$$\text{Hence,} \quad \begin{aligned} Z_o + Z_B &= 693 - \text{j}400 + 600 - \text{j}50, \\ &= 1{,}293 - \text{j}450. \end{aligned}$$

$$\text{Also} \quad Z_o - Z_B = 93 - \text{j}350.$$

$$\text{Thus} \quad \frac{Z_o + Z_B}{Z_o - Z_B} = \frac{1{,}293 - \text{j}450}{93 - \text{j}350},$$

$$\text{and} \quad \left|\frac{Z_o + Z_B}{Z_o - Z_B}\right| = \left(\frac{1{,}293^2 + 450^2}{93^2 + 350^2}\right)^{1/2},$$

$$\text{Thus} \quad 20 \log_{10} \left|\frac{Z_o + Z_B}{Z_o - Z_B}\right| = 10 \log_{10} \left(\frac{1{,}293^2 + 450^2}{93^2 + 350^2}\right),$$

$$= 10 \log_{10} 14{\cdot}3 = 15{\cdot}5.$$

Thus, $\underline{\text{balance return loss} = 15{\cdot}5 \text{ dB}}$

Q. **9.** *Describe how a fault on a low-loss transmission line may be located by plotting the input impedance/frequency characteristic and observing the regular pattern of maximum and minimum values.*

If the frequency separation between adjacent impedance maxima is 2 *kHz calculate the distance to the fault from the testing end. Assume the line to have a loop inductance of* 1 *mH/mile and a loop capacitance of* 0·15 *μF/mile.*

A. 9. Distance to the fault = 20·4 miles

See A. 3, Line Transmission C, 1965, *Supplement*, Vol. 59, No. 2, p. 47, July 1966.

Q. **10.** *Describe with the aid of schematic diagrams how power is provided for the equipment in a repeater station and explain what arrangements are made to ensure continuity of supply.*

How is electrical noise from the power plant prevented from interfering with the transmission circuits?

A. 10. The primary source of power is usually the commercial mains supply and there are two main methods of utilizing this for repeater station equipment:

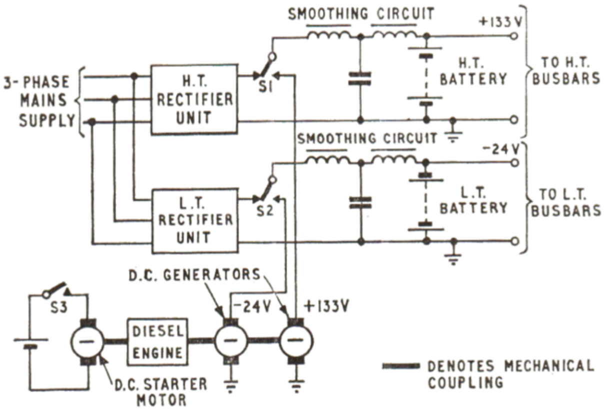
(*a*) By using floating batteries charged from the mains and distributing to individual racks by busbars, or,

(*b*) By feeding the mains to small power packs on individual racks to give a.c. for the valve heaters and rectified d.c. for the h.t. supply.

The essential requirements are good regulation and absolute reliability. The former is fairly easy to achieve because the load is constant (in contrast to the load in a telephone exchange) but the latter requires some form of alternative supply in case of mains failure. The alternative supply is usually provided by a prime mover such as an oil engine coupled to an alternator or d.c. generators.

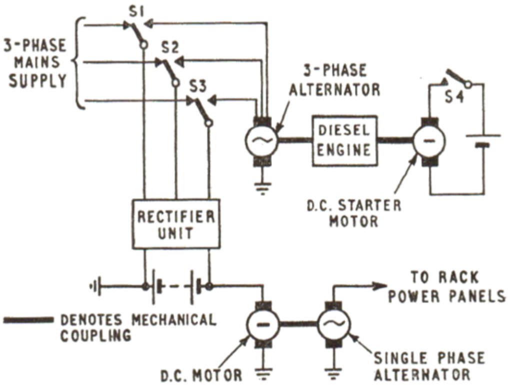
A typical arrangement for method (*a*) is shown in sketch (*a*). In the event of mains failure switch S operates to start the oil engine and the d.c. generators coupled to it take over from the mains rectifiers. No interruption occurs because the batteries continue to feed the equipment while the oil engine runs up to speed.

Sketch (*b*) shows a typical arrangement for method (*b*). An alternator is used to feed the individual power packs. This alternator is coupled to a d.c. motor running from the rectified mains supply backed up by a battery. In the event of mains failure, switch S operates to start the oil engine and the 3-phase alternator coupled to it takes over the supply. No interruption occurs because the d.c. motor continues to run from the floating battery while the oil engine runs up to speed.



(*a*)

Noise may enter the transmission circuits from the power supply itself if the smoothing arrangements are inadequate. With floating batteries the smoothing must be sufficient to suppress armature ripple from the generators and with individual power-packs it must be sufficient to suppress mains hum from the rectifiers. Mutual interference between transmission circuits may occur because of the use of common supplies and this can only be suppressed by making the internal impedance of the supply low enough to avoid trouble of this kind. The requirement is a very small impedance at signal frequencies and this is usually achieved by the use of decoupling capacitors.



(*b*)

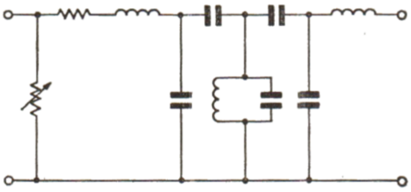
TELEGRAPHY C, 1967

Students were expected to attempt any six questions.

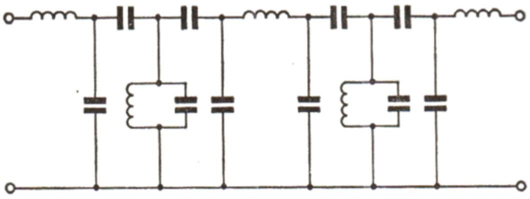
Q. **1.** *Sketch the circuit arrangements for the send and receive filters of a multi-circuit voice-frequency system. Explain why the designs of the two types of filter are different.*

What is the frequency spacing of the channels and why is this value chosen?

A. 1. Circuit arrangements of the components for typical send and receive filters are shown in sketches (*a*) and (*b*), respectively. Both are band-pass filters of the unbalanced type which results in economy of components.



(*a*)



(*b*)

The main function of the send filter is to restrict the sidebands of the modulated carrier so that they do not fall outside the frequency-range allocated for a given channel; the send filter also isolates any channel from the others as regards the keying input and the output load. The filter shown is a single-section band-pass filter. The variable shunt-impedance is to enable the channel output-power to be adjusted. The series impedance assists in preserving a constant input impedance.

The receive filter has to pass the frequency band appropriate to a given channel while drawing no appreciable energy at frequencies appropriate to the other channels. It comprises basically two band-pass sections in series; this sharpens the filter cut-off to exercise greater discrimination against unwanted frequencies and so reduces the effects of inter-channel interference. Compared with the attenuation at mid-band frequency, f_0, the attenuation at $f_0 \pm 35$ Hz is not more than 2·5 dB higher but at ± 120 Hz it is at least 30 dB greater.

The mid-frequencies of the channels are spaced at 120 Hz. This spacing enables 50-baud signals, with basic sidebands of $f_0 \pm 25$ Hz, to be transmitted with low characteristic distortion and with little inter-channel interference. It also permits higher modulation rates to be used if increased values of distortion and inter-channel interference can be tolerated.

Q. **2.** *Describe the difference in the basic design principles of distortion meters (TDMS) suitable for measuring:*

(*a*) *7-unit start-stop signals,*
(*b*) *7-unit synchronous signals.*

Include in your answer the timing chart for a typical distorted 7-unit start-stop signal and show how this signal would be displayed upon each type of meter.

Why is a 7-unit signal used in distortion measurements?

A. 2. (*a*) In a TDMS suitable for measuring start-stop distortion on 7-unit (or 7·5-unit) signals, the time-distortion of each element is essentially related to the commencement of the start element.

A cathode-ray oscilloscope is used with time-base circuits, the operation of which is initiated by the reception of the start element of a character. The primary time-base period is made equal to the nominal time of a unit signal-element (20 ms for a modulation rate of 50 bauds) and the cathode-ray horizontal traverse is repeated for each possible signal transition during the reception of a character (six traverses). The occurrence of a signal transition causes the beam to be intensified to produce a bright spot on the CRT screen. To permit observation of the distortion of the individual instants of modulation in a character, the six traces may be mutually displaced by a suitable vertical-deflexion circuit so that the trace forms a column of six lines. The repetition rate of the time base is controlled by an oscillator which is started when the start element of each character is received and stopped when the correct number of beam traverses has occurred.

To facilitate reading the distortion of the instants of restitution—which may be either advanced or retarded with respect to their nominal position—the commencement of the cycle is delayed by half a unit period so that undistorted signals are indicated at the centre of each traverse. Left and right of the central vertical position the scale is divided to show up to 50 per cent early and 50 per cent late distortion with reference to the ideal position of the spot.

On some designs of start-stop TDMS a spiral trace is used instead of the rectilinear traces for the purpose of spreading out the beam to correspond with the time of one character period.

(For a typical timing chart and display produced by a distorted start-stop signal on the horizontal trace see A. 2, Telegraphy B 1963, *Supplement*, Vol. 57, p. 21, Apl 1964, or, on a spiral trace, see A. 3, Telegraphy B, 1964, *Supplement*, Vol. 58, p. 9, A. 3, Apl 1965.)

(*b*) For measuring the distortion on 7-unit synchronous signals a cathode ray oscilloscope is used with a beam giving a circular trace, produced by a phase-splitting circuit. One revolution of the trace corresponds to the time of one signal element (20 ms at 50 bauds) and the trace develops a bright spot on the receipt of each signal transition. Ideally, with no distortion, all the spots from a train of signals would be superposed, the spot occurring at any point on the circle determined by the instant of arrival of a signal transition in relation to the instantaneous position of the rotary trace. When distortion is present the spots spread over a range of the trace according to the departure of the various instants of restitution from exact multiples of the nominal element duration. The circular scale can be marked with 100 divisions to show percentage distortion. (For a typical timing chart and display produced by a synchronous signal see A. 2, Telegraphy B, 1963. *Supplement*, Vol. 57, p. 21, Apl 1964.

Since the circular trace of the synchronous TDMS corresponds to the time of one signal element it is clear that the receipt of 7·5-unit signals would cause the point of appearance of the bright spot of an ideal signal to move to a new position 90° around the trace each time a new character was received. This makes it difficult to read distortion over a period corresponding to several characters.

For distortion measurements, particularly over V.F. channels, a repeated 7·0-unit start-stop signal is used. This signal can be read on a start-stop TDMS. Since it does not contain any half-element, when it is received on a synchronous TDMS the bright spots are ranged always around the same ideal position during any one observation. The standardized test-signal used consists of letter-shift, S, carriage-return, line-feed, Q, figure-shift, space, 9, each character having a stop signal of a single element duration, i.e. a total of 7·0 elements per character. This signal prints Q9S on a teleprinter, gives a range of mark/space ratios, provides 31 transitions for each repetition and the readings are all centred about the same point on the cathode-ray tube for a given observation.

Q. **3.** *An amplitude-modulated voice-frequency channel receiving equipment is suspected of being faulty. Describe what tests, measurements and readjustments might be made to the receiver to ensure satisfactory operation. State the purpose of the readjustments you describe.*

A. 3. The testing procedure will depend, to some extent, upon the type of v.f. equipment concerned. The circuit would first be taken out of service and replaced, if need, be by a spare circuit.

With the earlier equipment, comprising an amplifier valve followed by a rectifier valve, the channel lining-up procedure would be as follows. A tone (mark) signal would be requested from the distant station and a milliammeter (0–10 mA) plugged into the rectified-current jack which is in series with the anode circuit of the detector valve. The following tests would then be made:

(*i*) The test key of the receiver panel would be operated; this inserts a 10 dB attenuator into the input circuit of the amplifier-detector and also short-circuits the resistance component of the automatic-gain-control circuit to render this inoperative. The calibrated input potentiometer would then be adjusted until a rectified current of 3 mA was obtained. The potentiometer setting required to achieve this current should not exceed step 24.

(*ii*) With the test key restored to normal the rectified anode current should rise to a value between 4 and 8 mA.

(*iii*) With the input tone disconnected the rectifier anode current should not rise above 1·0 mA after one minute with the test key normal and should not exceed 0·5 mA when the test key is operated. The l.t. and h.t. voltages and the filament current may also be checked if necessary.

If the receiver does not pass test (*ii*) or (*iii*) the rectifier valve should be replaced by a new valve. Failure to pass test (*i*) may be due to either the amplifier or the rectifier valve being faulty but the amplifier valve should be changed first if tests (*ii*) and (*iii*) are passed satisfactorily.

After a valve has been changed the above tests should be repeated. It should then be possible to obtain the specified currents with a potentiometer setting between steps 15 and 21.

The distant station would then be requested to send 2 : 2 reversals at 50 bauds and a TDMS would be connected to the channel output. The relay-bias rheostat would then be adjusted, if necessary, until the distortion shows a minimum value. Alternatively, the output signals could be read on a milliammeter (35–0–35 mA) patched into the d.c. receive-wire circuit. Other test-signal combinations may then be requested from the distant station. With the Q9S signal the distortion should not exceed 8 per cent; if this limit is not achieved it may be necessary to replace the receive relay by a relay which has been cleaned and correctly adjusted.

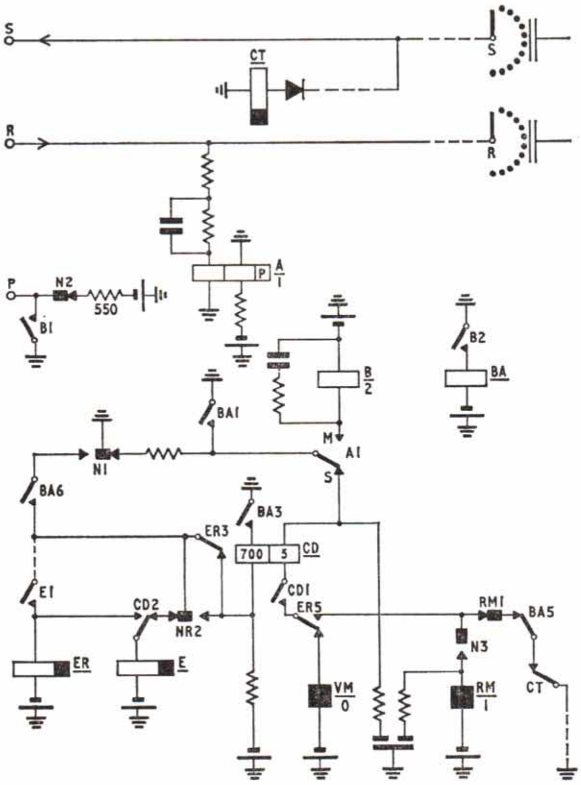
While the two stations are in co-operation in these tests it is usual for each station to check its receiver.

The above tests should ensure the replacement of any faulty valve, confirm the high insulation resistance of the valve grid circuits and the correct operation of the automatic gain control and also put the receiver at its optimum setting to withstand the effects of line-level changes.

With more modern equipment it may be possible to test the receiver in local, i.e. by connecting the receiver to its related send equipment after advising the distant station that the circuit will be temporarily out of service. After lining up the channel in local it is desirable to make the end-to-end distortion measurements described above.

Q. **4.** *Describe, with sketches of the circuit elements concerned, how the wipers of a final selector are stepped to the desired bank contacts. How are the wipers restored to normal when a connexion is cleared?*

A. 4. The last two digits of a subscriber's number are pulsed into the final selector. The circuits which control the positioning of the wipers are shown in the sketch.



When the final selector is seized the polarized relay, A, operates to the M contact due to the −80-volt call condition extended by the group selector over the R-wire. Relay B is operated via contact A1. Contact B1 provides a guarding earth potential to the P-wire and contact B2 operates relay BA. Contact BA1 covers the subsequent operation of N1. Relay CD is operated via contact BA3.

When the first pulse-train is dialled into the final selector, double-current signals actuate relay A. During each space period the vertical magnet is energized via contacts BA1, A1, relay CD (5-ohm winding), contacts CD1 and ER5, and steps the wipers to the contact-bank level corresponding to the digit dialled. During pulsing, relay B is energized during mark periods and sustained during space periods by the energy from the capacitor which shunts it. Relay CD is energized over its 5-ohm winding during the space periods, and sustained during mark periods by the current induced in its 700-ohm winding which is short-circuited via contacts N1, BA6 and ER3. The off-normal contacts, N, operate when the wiper shaft takes its first vertical step.

The interval between the two pulse-trains exceeds the release lag of relay CD which restores. Relay E is operated via contact CD2 and contacts N1, BA6 and NR2. Relay ER is operated via contacts E1, N1 and BA6. Relay CD is re-operated by contact ER3, breaking the short-circuit across its 700-ohm winding. Contact ER5 switches the pulsing path from the vertical to the rotary magnet.

The final pulse-train operates the rotary magnet during space elements and so steps the wipers to the contacts in the bank level corresponding to the units digit dialled. During pulsing, relays B and CD (via contact, NR2), are held operated as described above. The rotary off-normal contacts, NR, operate on the first rotary step.

When the calling subscriber sends a clear signal, contact A1 is moved to the space contact and relay B releases. All relays which have been held during the call are released due to the restoration of relay B, followed by relay BA. A circuit is completed by contact BA5 for the rotary magnet to drive forward and out of the bank via its interrupter contact, RM1. The wiper shaft is restored vertically by gravity and horizontally by a spring which was wound up during rotary stepping. The N and NR contacts restore to normal. Completion of the homing circuit may, however, be delayed by a contact of relay CT, which remains operated if the called subscriber's station has not cleared down. During the release period the selector is guarded against reseizure by contact N2 which delays restoration of the 550-ohm P-wire testing-battery.

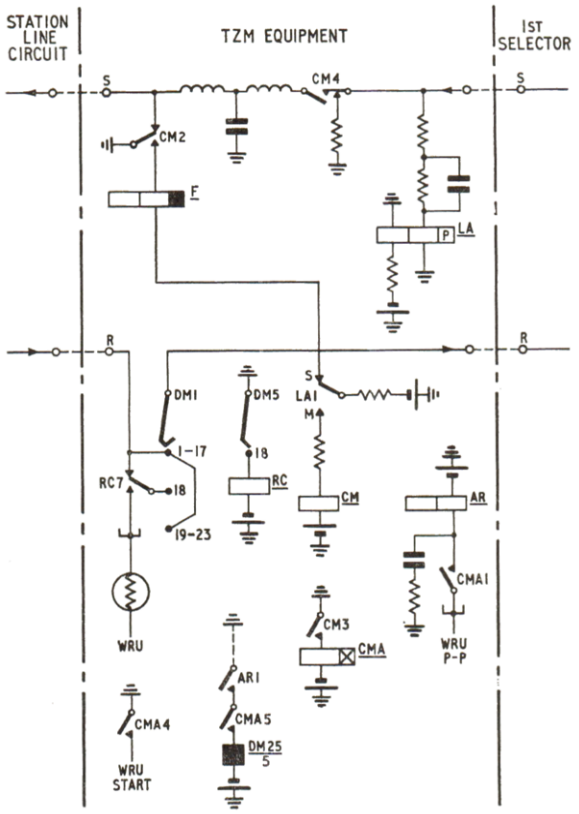
Q. 5. *Give a list of service signals which may be provided by a telex automatic exchange.*

Describe, with sketches of the appropriate circuit elements, how the "who-are-you" signal is sent from the exchange to the called subscriber's teleprinter.

A. 5. The service signals provided from a telex automatic exchange are OCC (subscriber engaged), NC (trunk busy), NP (spare line or level), DER (subscriber's line faulty), ABS (station shut down), MOM (awaiting reply from switchboard or service position).

The who-are-you (WRU) signal is sent from a time-zone metering equipment (TZM.), associated with the first selector, when the final selector has switched to the called station and the latter has returned the call-connected signal.

The TZM. equipment comprises a two-motion selector which controls metering on effective chargeable-calls, together with a sequencing uniselector, one of whose functions is to control the transmission of the WRU signal. The uniselector steps during dialling and stands on contact 17 (see sketch) when prepared to send the WRU signal. It is then stepped at 600 mS intervals by the WRU phasing pulses (PP)—these pulses are necessary to ensure the transmission of unmutilated start-stop signals corresponding to FIGS and WRU.



Receipt of the call-connected signal (−80 volts) from the called station over the S-wire operates relay LA to its M contact. Relay CM is operated via contact LA1. Contacts CM2–4 extend the call-connected signal back to the calling station. Relay CMA is operated via contact CM3. Contact CMA1 applies WRU phasing pulses to relay AR. The first subsequent phase pulse operates relay AR to operate the drive magnet, DM (via contacts AR1 and CMA5) which steps its wipers to contact 18 on termination of the pulse. At contact 18 relay RC is operated by the DM5 wiper. The DM1 wiper, via contact RC7, sends out the FIGS–WRU signal from the exchange pulse machine. A subsequent phase pulse steps the uniselector (via contact AR1) to contact 19 to disconnect the WRU signal from the R-wire.

Q. 6. *Describe, with sketches of the circuit elements, how on subscriber-dialled telex calls the calling subscriber's meter is caused to register the appropriate fee, depending upon the destination and duration of the connexion.*

A. 6. See A. 8. Telegraphy C, 1964, *Supplement*, Vol. 58, p. 40, July 1965.

Q. 7. *Sketch a block-schematic diagram and describe the operation of a typical push-button tape-relay system. What is the function of each type of start-stop machine used in this system?*

A. 7. For an answer to the first part of the question see A. 8. Telegraphy C, 1965, *Supplement*, Vol. 59, p. 45, July 1966.

Printing Reperforator. This is required for each incoming channel to deliver and store a received message in the form of perforations in a paper tape. In order that an operator can read the address and the prefix instructions the message is also printed upon the tape either over the "lids" of the perforations in chadless tape, or on more modern machines in the spaces between the central feed-holes. Use of the perforated tape enables an incoming message to be despatched over an outgoing channel without the need for an operator to retransmit the message by hand from a keyboard.

Automatic Transmitter. By placing the perforated tape into the automatic transmitter a message can be transmitted at the maximum speed of the system without further manual intervention. An electrically-operated clutch in the transmitter couples up the tape feed when the channel becomes free; the transmitter reads the holes in the tape and sends corresponding start-stop signals. By means of contacts controlled by the presence of the tape a supervisory signal is automatically produced when transmission of a tape is completed and the transmitter is brought to rest.

Serial-Number Transmitter. This machine, essentially a special form of automatic transmitter, transmits a serial number in the prefix of every message before transmission in order to provide a safeguard against a lost message. The transmitter advances the serial number by one for each message and it is usually reset to zero at midnight.

Teleprinter. A teleprinter is used to monitor all messages passing through the centre in order to keep a file record for such purposes as answering queries, accounting, etc.

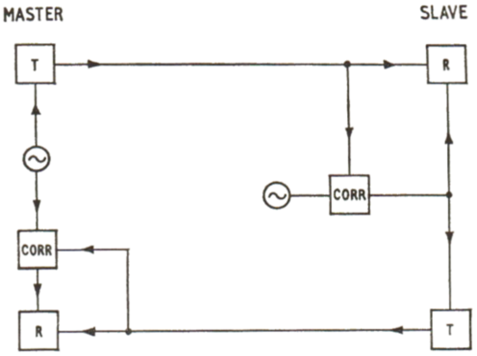
Q. 8. *Describe with the aid of a block diagram the general principles of the ARQ error-correcting radio-teleprinter system.*

Describe in detail how correct synchronism and phase are established and maintained between the two stations.

A. 8. For the first part of the answer see A. 10, Telegraphy C, 1961, *Supplement*, Vol. 55, p. 44, Oct. 1962.

In a synchronous ARQ system the two co-operating stations are each controlled from local oscillators having the same nominal frequency and a stability better than 1 part in 10^6; even so, after a period of about 30 minutes two stations could be significantly out of step.

For purposes of element synchronizing and character phasing, one station is designated the master station and the distant station becomes the slave station (see sketch). At the master station, the transmitter is controlled solely by the local oscillator; its receiver is also driven by the same local oscillator but means are provided for correction by the incoming signal transitions. At the slave station, the transmitter and receiver are locked together, but with a delay of several elements in the transmitter timing compared with the receiver. The reason for this is so that the master transmitter is not presented simultaneously with a traffic character and an error correcting (RQ) signal.



To effect element synchronization, pulses generated at the element rate by the local oscillator are compared with the transitions of the incoming signals. Differences in timing of the local pulses with respect to the incoming transitions, either early or late, are integrated on a 13-position differential electronic counter which will cause a synchronizing step (equivalent to 1 per cent of an element) each time a count of six is obtained; the counter then resets itself to the centre position for a fresh count. The effect of synchronizing is to ensure correspondence or identity between incoming elements and locally-generated elements to achieve maximum receiving margin.

Once correctly set up, an ARQ system will normally remain in correct character phase. If a system falls out of phase, for example due to loss of the radio path for a time in excess of that covered by the oscillator stability, the out-of-phase condition will be shown up by continuous RQ cycling, even when the radio path is restored. Phasing is restored by retarding the timing of a corrected equipment one element at a time for each received error until the RQ cycling ceases. Correction may be applied manually by a press-button one step at a time or automatically, according to the equipment design.

Q. 9. *Give reasons why cable-code systems are gradually being superseded for submarine telegraphy. What methods are taking the place of cable-code systems and with what advantages?*

A. 9. Several reasons have contributed to the gradual supersession of the cable-code system. For many services, methods involving re-transmissions with manual handling are giving way to through-transmission systems, using either circuit switching or message switching. For such systems it is highly desirable or even essential that the switched or interconnected circuits should work at the same speed and use the same code. The teleprinter five-unit code at a modulation rate of 50 bauds (6·6 characters per second) is the standardized system. Cable-code systems cannot be readily integrated into such networks. Regarding their own disadvantages, cable-code systems have given valuable service over a long period of time but by present standards their speed is low due to the characteristics of the d.c. cables on which they operate. Furthermore, the apparatus, consisting of special automatic transmitters, regenerative repeaters and direct-receiving printers, is highly complex and costly to maintain and to purchase, being specialized equipment required only in small quantities.

The gradual spread of inter-continental submarine telephone-cables equipped with submerged repeaters has largely usurped the field previously held by cable-code systems. Using standard multi-circuit voice-frequency telegraph equipment, 22 or 24 duplex 50-baud circuits of low distortion can be provided from each 3 kHz bearer circuit on a telephone submarine-cable: the number of telegraph circuits can be doubled if need be by the addition of time-division equipment at the telegraph-terminal stations. The field of intercontinental communication is now being shared by satellite communication systems capable of yielding large groups of stable telegraph circuits using the same telegraph-terminal equipment that has been standardized for land-line and submarine-cable operation. For all these reasons the cable-code system is no longer economic.

Q. 10. *Describe in some detail the process of receiving a positive photographic print by photo-telegraphy. If a subsequent picture is to be received as a negative upon film, what changes in adjustment are necessary at the transmitter or receiver?*

A. 10. See A. 5, Telegraphy C, 1966, *Supplement*, Vol. 60, p. 35, July 1967.

COMMUNICATION RADIO C, 1967

Students were expected to answer any six questions.

Note on A. 5. Radio C, 1966, *Supplement*, Vol. 60, No. 2, p. 47, July 1967.

Attention is drawn to the fact that the British Standards Institution have revised their earlier definition of modulation index, the current definition being as follows:

Modulation Index: In frequency modulation, with a sinusoidal modulating wave, the ratio of the frequency deviation to the frequency of the modulating wave.

Using this definition in Question 5, the amplitude of the modulating wave is half the maximum permissible amplitude and the frequency deviation is therefore 75/2 = 37·5 kHz. The frequency of the modulating wave is 1,500 Hz and the modulation index is therefore,

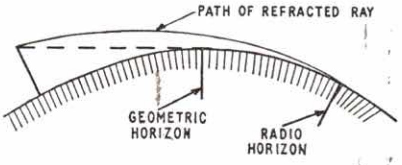
$$m = \frac{37\cdot 5}{1\cdot 5} = \underline{25.}$$

Q. 1. *Explain why v.h.f. radio signals can be received beyond the line of sight distance from the transmitter.*

Why is an effective radius equal to 4/3 times the actual radius of the earth often assumed when considering propagation paths at v.h.f.?

Show that, under normal propagation conditions, the distance in miles to the radio horizon is approximately equal to $\sqrt{2h}$ where h is the height of the aerial in feet. (The radius of the earth may be taken as 4,000 miles.)

A. 1. The refractive index of the lower atmosphere at radio frequencies (the radio refractive-index) depends on the air pressure, temperature and amount of water vapour. Under normal conditions the radio refractive-index decreases slowly with height. This causes a v.h.f. radio wave to be continuously refracted so as to follow a path which curves slightly downwards towards the earth. The radio horizon, therefore, lies somewhat beyond the optical or geometric horizon, as shown in sketch (*a*).

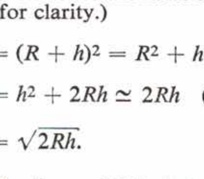


(*a*)

V.h.f. radio waves penetrate into the "shadow" zone, beyond the range of the direct ray, because the wave is diffracted round the curved surface of the earth. In the shadow zone, the field strength falls more rapidly with distance. Some energy from the transmitter is also scattered into the shadow zone from the small irregularities and random variations of radio refractive-index which occur in the atmosphere.

Occasionally, strong fields can be received at considerable distances by means of duct propagation. This occurs when abnormal atmospheric conditions cause the radio refractive-index to decrease more rapidly with height, resulting in the formation of a duct through which the radio wave can propagate, with relatively small loss, by successive reflexions from the boundary surfaces. Propagation of frequencies below about 100 MHz to even greater distances may occasionally occur by reflexion from the ionosphere due to the effect known as Sporadic E. This results from the occasional presence of small areas of abnormally high ionization density in the E region.

Under normal conditions, the curvature of the direct-ray path, due to refraction in the atmosphere, can be allowed for by assuming the wave travels in a straight line above an earth of increased radius. The curved-ray path above the actual earth is then equivalent to a straight path above an earth of radius increased by a factor of 4/3. In considering path profiles it is convenient to make these assumptions since it is then easier to estimate the clearance of the path above the terrain.



(*b*)

In sketch (*b*), *R* is the equivalent radius of the earth, *h* is the height of the aerial and *d* is the distance to the radio horizon. (The dimension *h* is greatly exaggerated for clarity.)

$$R^2 + d^2 = (R + h)^2 = R^2 + h^2 + 2Rh$$

$$\therefore \quad d^2 = h^2 + 2Rh \simeq 2Rh \quad (\text{Since } h \ll 2R).$$

$$\therefore \quad d = \sqrt{2Rh}.$$

Taking $R = 4/3 \times 4{,}000$ miles, and *h* in feet,

$$d\,(\text{in miles}) = \sqrt{\frac{2 \times 4 \times 4{,}000}{3 \times 5{,}280}\,h}$$

$$= \sqrt{\frac{32}{15\cdot 9}\,h}$$

$$\therefore \quad \underline{d \simeq \sqrt{2h}.}$$

Q. 2. *Write brief notes on the various sources of noise that arise in a radio receiver.*

Explain what is meant by the terms

(a) noise factor of a radio receiver,
(b) equivalent-noise resistance of a valve.

A. 2. Noise can arise in a radio receiver from the following sources:

(*a*) *Thermal noise.* The free electrons in a conductor are in a state of random motion due to thermal agitation, which generates a random noise voltage in any impedance having a resistive component. This voltage increases with the temperature above absolute zero, and its mean square value is given by the Nyquist formula,

$$E_n^2 = 4\,k\,T\,B\,R,$$

where,
k = Boltzmann's constant ($1{\cdot}37 \times 10^{-23}$ joules/°K),
T = absolute temperature (degrees Kelvin),
B = bandwidth (Hz),
R = resistance (ohms).

In a radio receiver, the thermal noise most likely to be significant is that associated with the input circuit of the first stage.

(*b*) *Shot noise.* The current in a valve consists of the flow from cathode to anode of a very large number of electrons, each of which carries a small electrical charge. Because of the random manner in which individual electrons are emitted by the cathode and arrive at the anode, the instantaneous current fluctuates slightly about its mean value over any small interval of time. This random variation is shot noise, and contains energy extending uniformly over a very wide frequency range. Shot noise is greater in the absence of space charge, which partly smooths the variations. Shot noise also arises in transistors due to similar random variations in flow of current carriers across the junctions.

(*c*) *Partition noise.* In valves having more than one current collecting electrode, the shot noise is increased by partition effect. This arises from small random variations in the way the current divides between the electrodes. In transistors, a similar effect occurs in the division of emitter current between collector and base.

(*d*) *Induced grid noise.* The flow of electrons past the control grid induces on it a randomly-fluctuating noise-voltage which is amplified by the valve. The noise voltage is related to the grid-cathode input conductance resulting from transit-time effect and increases with frequency, becoming significant above about 20 MHz.

(*e*) *Flicker-effect noise.* Flicker noise, or $1/f$ noise, is inversely proportional to frequency and can be of importance below 1 kHz. In valves it is attributed to low-frequency variations arising from irregularities of cathode emission. It occurs also in transistors and is probably due to surface and leakage effects.

Additional noise can arise in valves from secondary emission at the anode, and from collision ionization of residual gas molecules when struck by electrons in transit. Other types of noise in receivers include microphony caused by mechanical vibration, and power-supply hum due to imperfect smoothing or screening. Resistor noise, which is additional to thermal noise, is due to non-linear effects in resistors and depends on their construction and operating conditions; for example, carbon composition resistors carrying d.c. are noisier than carbon film or wire-wound resistors of the same value. In a well-designed receiver it is the first-stage amplifier that contributes most of the noise arising in the receiver. Below about 20 MHz the atmospheric noise picked up by the aerial generally exceeds noise arising in the receiver.

(*a*) *Noise Factor* indicates the extent to which the noise generated by the receiver degrades the signal-to-noise ratio available at the input, and is given by

$$\text{Noise Factor, } F = \frac{\text{Input signal-to-noise power ratio}}{\text{Output signal-to-noise power ratio}}.$$

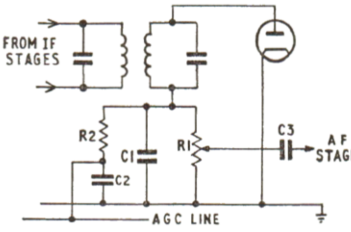
The input noise-power is due to thermal noise in the resistance of the signal source, and noise factor is therefore also equal to the ratio of the total output noise-power to that part of it which originates in the source resistance. The noise factor can also be stated as the output signal-to-noise ratio of the equivalent ideal noiseless receiver over that of the actual receiver.

(*b*) The random noise generated by a valve under specified operating conditions can be referred to its control grid by assuming the valve to be noiseless and an equivalent noise voltage applied to the grid. The *equivalent noise resistance* of the valve is the value of grid resistance in which thermal noise at room temperature equals this equivalent grid noise-voltage. In this way, the noise performance of the valve can be expressed independently of the circuit bandwidth.

Q. 3. *Describe a method of automatic gain control (a.g.c.) of a radio receiver, and draw circuit diagrams showing how the control voltage is derived and how it is applied to control the gain of an amplifying stage. Explain what is meant by delayed a.g.c.*

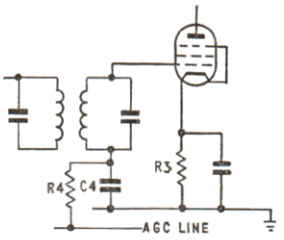
The output of a receiver is required to vary by not more than 6 dB for a 60 dB change of input signal. Three amplifying stages are controlled in each of which the gain varies by 3 dB/V of grid bias. Calculate the delay voltage, assuming the a.g.c. voltage is directly proportional to the amplitude of the carrier.

A. 3. The usual method of a.g.c. is to derive a d.c. control voltage from the signal at the detector and apply it so as to increase the negative grid-bias of the variable-mu valves used in the r.f. and i.f. stages. A small increase of carrier voltage at the detector can thus be made to produce a large reduction of receiver gain. Overloading is then prevented and variations of input signal level (e.g. due to fading) are considerably reduced in the output.



(*a*)

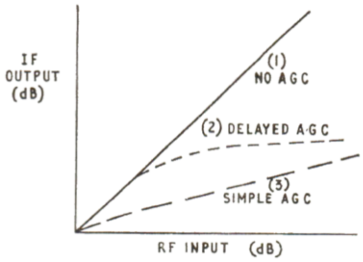
In the circuit shown in sketch (*a*), the amplitude-modulated signal from the i.f. amplifier is rectified by the diode, which has a series load consisting of the parallel combination of capacitor C1, and resistor R1. The i.f. is by-passed by capacitor C1, and the voltage across the load contains a d.c. component proportional to the carrier level and an a.f. component corresponding to the modulation. The a.f. component is passed on to the a.f. amplifier through capacitor C3, which blocks the d.c. The resistor, R2, is large compared with the reactance of capacitor C2 at the lowest audio frequency so the d.c. component alone appears on the a.g.c. line.



(*b*)

A method of applying the control voltage to the grid circuit of an i.f. stage is shown in sketch (*b*). The cathode resistor R3, provides normal grid-bias. The control voltage is fed through resistor R4, which decouples the circuit in conjunction with capacitor C4.

The simple a.g.c. described above has the disadvantage that the receiver gain is reduced by the control voltage even when weak signals are being received, see curve 3 of sketch (*c*). To overcome this, a fixed or "delay" bias is inserted in the diode circuit and prevents any control voltage being developed unless the signal voltage at the detector is greater than a predetermined value. The full gain of the receiver is then available for all signals less than this value, as indicated by curve 2 of sketch (*c*). Normally, the delay bias inhibits a.g.c. action until the signal level is large enough to produce the standard a.f. output.



(*c*)

In the problem, when the input rises by 60 dB and the output by 6 dB, reduction of gain = 60 − 6 = 54 dB, degree of control = 3 × 3 = 9 dB/volt.

$\therefore$ increase of control voltage = 54/9 = 6 volts.

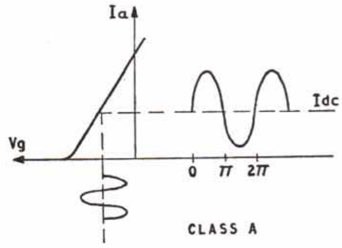
Since the control voltage equals the peak value of carrier voltage at the detector less the fixed delay-bias, the peak carrier-voltage must also increase by 6 volts. But the carrier level at the detector increases by 6 dB, i.e. by a factor of 2 so, to meet the stated condition, its peak value must initially be 6 volts and will increase to 12 volts when the input is raised by 60 dB. With the a.g.c. operative, the ratio of output change to input change decreases as the signal level is raised, and is therefore greatest when the delayed a.g.c. action just commences. Since, in the problem, the output change for a 60 dB input change is not to exceed 6 dB, the a.g.c. action should commence when the peak carrier-voltage at the detector is 6 volts. The required value of delay bias is therefore 6 volts.

Q. **4.** *Briefly explain the differences between Class-A, Class-B and Class-C amplification, and by reference to a typical anode-current/grid-voltage characteristic show the anode-current waveforms for each case if the input signal is sinusoidal. If the cut-off voltage is* −10 *volts, state typical values of grid bias required for each of the three conditions.*

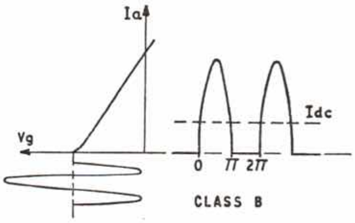
A triode valve having an anode a.c. resistance of 500 *ohms and an amplification factor of* 5 *is connected in parallel with one having an anode a.c. resistance of* 1,000 *ohms and a mutual conductance of* 8 *mA/V. For the combination calculate*

(*a*) *the anode a.c. resistance,*
(*b*) *the mutual conductance,*
(*c*) *the amplification factor.*

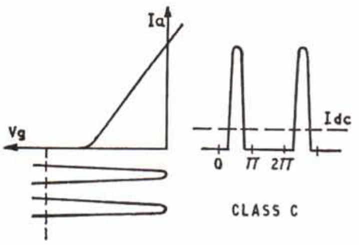
A. 4. *Class A.* Anode current flows over the whole cycle, the angle of flow being 360°. The grid is biased half-way between zero volts and the anode-current cut-off voltage. The anode efficiency is relatively low, the theoretical maximum being 50 per cent, but practical values are more usually 20 to 40 per cent. Class A provides linear amplification and is widely used for small-signal voltage amplifiers and low-power applications where linearity is more important than efficiency.



Class B. Anode current flows for half the cycle, the angle of flow being approximately 180°. The grid is biased nearly to cut-off. The theoretical maximum efficiency is 78·5 per cent, and in practice about 60 per cent may be obtained. Two valves operating in push-pull are often used in audio-frequency power applications to give low distortion and good efficiency. A single-ended stage gives linear amplification over half the cycle and can be used as an r.f. power amplifier for amplitude-modulated waves.



Class C. Anode current flows for less than half the cycle, the angle of flow being less than 180°. The grid bias depends on the driving-signal voltage and the angle of flow required and is typically about twice the cut-off voltage. Normally grid current flows at the peak of the cycle. Because of the small angle of flow, efficiency is high and up to about 85 per cent may be obtained. Class C is used in tuned r.f. power-amplifiers and harmonic generators. Its non-linear operation makes it unsuitable for a.m. waves, except when used in conjunction with a high-power anode modulator. Class C provides high-efficiency, high-power amplification of c.w. and f.m. waves.



If the cut-off voltage is −10 volts, typical values of grid bias would be −5, −10 and −20 volts for the three classes respectively.

For the first valve, $r_{a1} = 500$ ohms, $\mu_1 = 5$,

$$g_{m1} = 5/500 \times 10^{-3} = 10 \text{ mA/volt.}$$

For the second valve, $r_{a2} = 1{,}000$ ohms, $g_{m2} = 8$ mA/volt.

$$\mu_2 = 1{,}000 \times 8 \times 10^{-3} = 8.$$

For the combination,

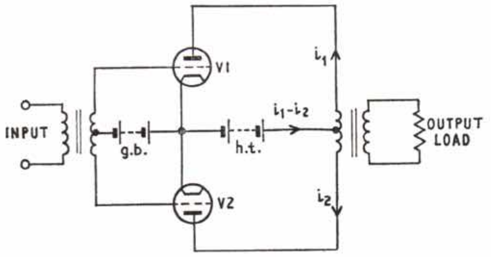
$$(a) \quad r'_a = \frac{r_{a1}r_{a2}}{r_{a1}+r_{a2}} = \frac{500 \times 1{,}000}{1{,}500} = \underline{333 \text{ ohms.}}$$

$$(b) \quad g'_m = g_{m1} + g_{m2} = 10 + 8 = \underline{18 \text{ mA/volt.}}$$

$$(c) \quad \mu' = g'_m \times r'_a = 18 \times 10^{-3} \times 333 = \underline{6.}$$

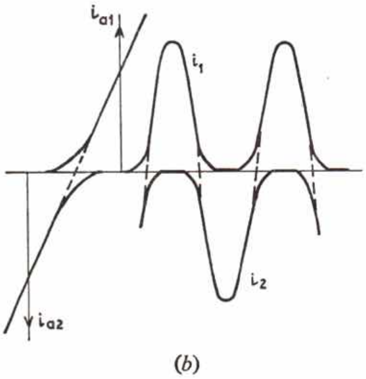
Q. **5.** *Draw a circuit diagram and explain the action of a Class-B push-pull amplifier for audio-frequency power amplification. Show by means of a sketch how linear amplification is obtained when the valves are biased to their projected cut-off point.*

A. 5. The circuit of a class-B push-pull audio-frequency power-amplifier as shown in sketch (*a*) uses two closely-matched triodes with centre-tapped input and output transformers. The grid-bias supply is common to both valves and biases the grids nearly to anode current cut-off. The signal voltages applied to the grids are of equal amplitude and of opposite phase. Each valve draws anode current during the half-cycles when its grid is positive with respect to the centre tap (i.e. when the grid-cathode voltage is less than the cut-off value), and is cut-off for nearly the whole of the negative half-cycle. The valves thus conduct alternately to amplify successive half-cycles of the signal.



(*a*)

The anode currents i_1 and i_2 flow through the primary of the output transformer in opposite directions so that the current in the load connected across the secondary is proportional to the difference $i_1 - i_2$. The load current therefore follows both positive and negative half-cycles in their correct sense, giving a true reproduction of the input signal. Because the d.c. components of the anode currents are equal and opposite in the primary of the output transformer, there is no resultant d.c. magnetization of the transformer core so maximum inductance is obtained and a smaller transformer can be used. Second harmonic and other even-order distortion products due to the curvature of the valve characteristics also cancel in the output so that high output-power can be obtained with low distortion. It is necessary for the valves to be selected as a matched pair having as nearly as possible identical characteristics. The class-B mode of operation provides high anode efficiency, of the order of 60 per cent. Moreover, with no input signal, only a low power is consumed since the static anode currents are small. The h.t. current varies with signal strength and an h.t. supply having good voltage regulation is therefore required.



(*b*)

Sketch (*b*) shows the combined dynamic characteristic of the two valves when biased to projected cut-off, together with the grid-voltage and anode-current waveforms for a sine-wave input. The projected cut-off point lies about half way along the curved part of the characteristic and is determined by extending the main linear part of the characteristic to intersect the grid-voltage axis. The lower characteristic in the sketch is inverted in order to show the resultant currents more clearly. The resultant combined characteristic is practically linear if the valve characteristics are closely matched and the bias point is correctly chosen.

Q. 6. *Derive an expression for the output power of a double-sideband amplitude-modulated transmitter in terms of the unmodulated carrier output power and the depth of modulation.*

The output power of a radio transmitter is 1 *kW when modulated to a depth of* 100 *per cent. If the depth of modulation is reduced to* 50 *per cent what is the power in each sideband?*

A. 6. For the amplitude-modulated wave let the carrier voltage be $V_c \sin \omega t$, and the modulating voltage be $V_s \sin pt$. The depth of amplitude modulation, m, is then equal to V_s/V_c, and the instantaneous voltage of the modulated wave is given by

$$\begin{aligned} v &= V_c \sin \omega t(1 + m \sin pt) \\ &= V_c \sin \omega t + mV_c \sin \omega t \sin pt \end{aligned}$$

By expansion,

$$v = V_c \sin \omega t + \frac{m}{2} V_c \cos (\omega - p)t - \frac{m}{2} V_c \cos (\omega + p)t.$$

These three component terms are respectively the carrier, the lower-sideband frequency, and the upper-sideband frequency. The peak voltage of each sideband frequency is $\frac{m}{2}$ times the peak carrier-voltage, and the power in each sideband frequency is therefore $\frac{m^2}{4}$ times the carrier power. The power of the modulated wave is the sum of the powers of the three components. If the carrier power is P_c, the total power of the modulated wave is

$$P_t = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c.$$

$$\therefore \quad \underline{P_t = P_c \left(1 + \frac{m^2}{2}\right)}.$$

For 100 per cent modulation depth, $m = 1$ and $P_t = 1{,}000$ watts.

$$\therefore \; 1{,}000 = P_c(1 + \tfrac{1}{2}) \text{ and } P_c = \frac{2{,}000}{3} = 667 \text{ watts.}$$

From the expression derived above, the carrier power, P_c, is independent of the depth of modulation m, and the power in each sideband is $\frac{m^2}{4} P_c$. If the depth of modulation is reduced to 50 per cent the power in each sideband becomes

$$P_s = \left(\frac{0{\cdot}5}{4}\right)^2 \times 667 = \underline{41{\cdot}67 \text{ watts.}}$$

Q. 7. *Explain the meaning of the following terms in relation to frequency-modulation:*

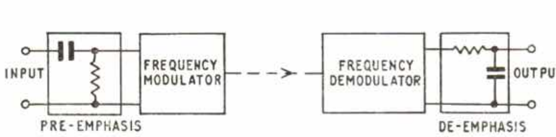
(*a*) *frequency deviation,*
(*b*) *modulation index,*
(*c*) *deviation ratio.*

What is the reason for the use of pre-emphasis in an f.m. system? Why is a limiter commonly used in an f.m. receiver?

A. 7. The *frequency deviation* is the peak difference between the instantaneous frequency of the modulated wave and the carrier frequency, in a cycle of modulation. Frequency deviation is directly proportional to the amplitude of the modulating signal.

The *modulation index* (in f.m. with a sinusoidal modulating wave) is the ratio of the frequency deviation to the frequency of the modulating wave.

The *deviation ratio* of an f.m. system is the ratio of the maximum permissible frequency-deviation to the maximum frequency of the modulating signal.



Pre-emphasis is used in f.m. systems in order to improve the signal-to-noise ratio. As shown in the sketch the pre-emphasis network is inserted before the modulator and attenuates low frequencies more than high frequencies. The relative amplitudes of the higher frequency components of the signal applied to the modulator are thus increased and produce larger deviations of the carrier than they would without pre-emphasis. At the receiver, noise and interference produce unwanted frequency deviations of the carrier which increase in proportion to the frequency difference between the disturbance and the carrier frequency. Consequently, if noise is uniformly distributed over the i.f. bandwidth then noise amplitude at the output of the demodulator increases progressively with frequency over the output-signal bandwidth. The de-emphasis network, which follows the demodulator, has a frequency characteristic that is the inverse of that of the pre-emphasis network. This attenuates the higher frequency components of both the noise and the signal at the output of the demodulator. The signal is thereby restored to normal (because of the pre-emphasis applied at the transmitter) but the noise is attenuated, thus giving an overall signal-to-noise improvement.

The purpose of the limiter in an f.m. receiver is to remove amplitude modulation and deliver a signal of constant amplitude to the discriminator. The use of a limiter is necessary when a discriminator is employed to demodulate the signal as the discriminator responds to amplitude variations as well as frequency variation of the signal. Any amplitude variations are unwanted in f.m. and they would appear in the output as distortion and noise. If the input signal is sufficient to fully drive the limiter its output is independent of input level and all amplitude variations are removed.

Unwanted amplitude modulation of the f.m. wave can arise in the transmitter or in the receiver (due, for example, to non-uniform response over the signal bandwidth). Slower variations of signal strength can result from propagation effects such as fading or reflections from aircraft. The limiter suppresses amplitude modulation and eliminates all but very deep fades, thus providing a very effective automatic-gain-control. Noise and interference superimposed upon the signal produce unwanted modulation of both amplitude and phase. The effect of such disturbances is reduced by the limiter action since the amplitude variations are suppressed and only the unwanted phase variations will remain to contribute to the output noise.

Q. 8. *Draw the circuit diagram and explain the action of either*

(*a*) *a Foster–Seeley discriminator or*
(*b*) *a ratio detector.*

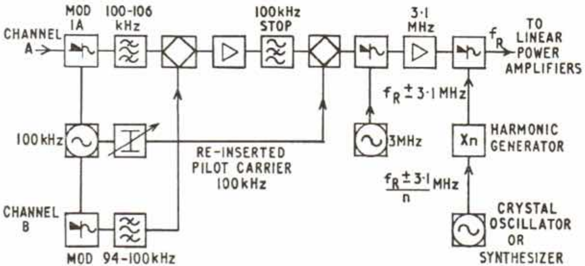
If an interfering c.w. signal having an amplitude of 10 *per cent of the carrier is present at the input of an f.m. receiver what is the phase deviation produced by the interference? What is the resulting frequency deviation when the frequency separation of the signals is* (*a*) 1 *kc/s* (*kHz*), (*b*) 10 *kc/s* (*kHz*)?

A. 8. The frequency deviations are as follows:

(*a*) 100 Hz. (*b*) 1,000 Hz.

Q. 9. *Draw the block diagram of a high-power short-wave transmitter capable of transmitting two independent telephony sidebands with reduced carrier. Briefly outline a method of checking the linearity of the transmitter.*

A. 9. Sketch (*a*) shows a simplified diagram of the equipment for the assembly of two audio-channels each having a bandwidth of approximately 100 to 6,000 Hz and their modulation as independent sidebands of the radio-frequency carrier. In practice, each of these channels might comprise two telephony-channels of about 3 kHz bandwidth assembled by audio-frequency modulating equipment and connected to the input of the i.s.b. drive equipment shown in sketch (*a*).

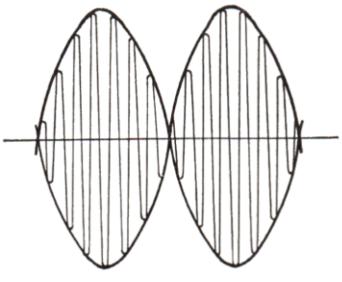


(*a*)

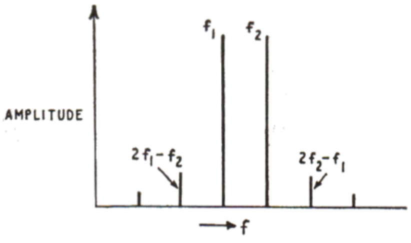
The audio inputs A and B each modulate a 100 kHz carrier frequency, and channel filters having sharp cut-off characteristics select the upper sideband from MOD 1A and the lower sideband from MOD 1B. The outputs from these filters are combined, and a very narrow bandstop filter is used to suppress the 100 kHz carrier component, leaving only the two independent sidebands. A separate output from the 100 kHz source then supplies the pilot carrier at a reduced and precisely-determined constant level for re-insertion in the signal path as shown. The pilot carrier is transmitted with the signal and is used in the receiver for a.g.c., and if necessary a.f.c.

Translation of the combined signal from 100 kHz to the final radio frequency is accomplished in two stages: the MOD 2, in which the signal modulates a 3 MHz carrier and the resulting upper sideband at 3·1 MHz is selected; and the MOD 3, which is supplied with a radio-frequency carrier differing by 3·1 MHz from the required radiation frequency. The r.f. carrier supply for the MOD 3 may be obtained from harmonic-generator stages driven by a crystal-controlled master oscillator or a frequency synthesizer. The transmitter power-amplifiers which follow must operate in a linear mode to avoid distortion of the signal.

A method of checking the linearity of the transmitter is to apply two audio-frequency tones of equal amplitude, spaced about 1 kHz apart, to one of the channel inputs. A monitor receiver loosely coupled to the transmitter output translates the signal to an intermediate-frequency where the waveform may conveniently be examined on an oscilloscope. Non-linearity will show as a distortion of the sine-wave beat-frequency pattern which should be obtained as shown in sketch (*b*). A more sensitive method is to measure the amplitudes of the third-order intermodulation products by means of a spectrum analyser. If the test-tone frequencies are f_1 and f_2, the third-order products caused by non-linearity of the amplifier will be at frequencies $(2f_1 - f_2)$ and $(2f_2 - f_1)$, as shown in sketch (*c*). In a high-quality transmitter the amplitudes of these products should not be greater than about 35 dB below the amplitude of either of the two equal-amplitude test frequencies.



(*b*)



(*c*)

Q. **10.** *Explain clearly what is meant by the gain of an aerial.*

Sketch and describe a rhombic aerial suitable for h.f. transmission. What are the main design features that determine its performance? State typical values for (*a*) *the gain,* (*b*) *the frequency bandwidth, and* (*c*) *the angle of elevation of the main lobe.*

TELEPHONY C, 1967

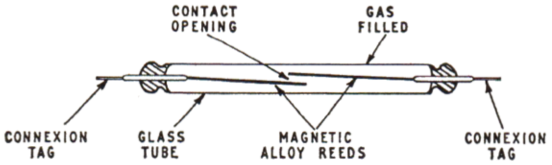
Students were expected to answer any six questions

Q. **1.** *Sketch and describe the construction and principle of operation of a dry-reed relay.*

List the desirable characteristics for a switching element in the speech path in a telephone switching system and discuss the relative merits for the function of:

(*a*) *a two-motion selector wiper and bank contact,*
(*b*) *a dry reed contact,*
(*c*) *a transistor.*

A. 1. A typical dry-reed relay, used in telephone exchange development, consists of a cylindrical coil surrounding dry reed inserts, usually four or six in number. Each insert comprises a glass tube, into each end of which is sealed an accurately positioned magnetic alloy reed or blade. The simplest and most usual form is a make-action unit as shown in the sketch. The inner ends of the reeds overlap, and in the unoperated state are separated by a gap of 5–10 mils.



The overlapping portion of each blade, which forms the contact area, is usually gold plated, great care being taken in manufacture to produce a contact free from impurities and mechanical defects. Each blade consists of a length of 50 : 50 nickel–iron wire, which has been pressed to give a flat cross-section at each end. The outer end may be used as a soldering tag. To prevent contamination and corrosion by the atmosphere, the tube is filled with either hydrogen or nitrogen, or a mixture of both.

When a current is passed through the surrounding coil, in either direction, the blades will become magnetized in such a way that the two adjacent ends forming the contact will be of opposite polarity. If the current is sufficient, the blades will be mutually attracted, making electrical contact. On cessation of the current, the blades will separate due to their flexure.

The desirable characteristics of a switching element to be used in the speech path of a telephone system are,

(*a*) low-on and high-off resistance,
(*b*) electrical parameters that are constant with temperature,
(*c*) freedom from noise,
(*d*) fast operate and release,
(*e*) small size,
(*f*) long contact life,
(*g*) high reliability of operation,
(*h*) low cost.

(*a*) The two-motion wiper and bank contact is an economical method of speech circuit switching in a step-by-step exchange. It is a useful method of providing a reliable one-input, multi-output switch, the contact resistance being adequately low if well maintained. However, as a device for new designs of common control exchange, it has disadvantages compared with other devices. Without considerable maintenance effort the contact can be too noisy for some purposes. The speed of operation is low. The device does not lend itself to mass production and assembly to the same extent as do other items.

(*b*) The dry-reed relay is an ideal method of utilizing a precious metal contact, in that the deleterious effects of the atmosphere are excluded. It has fast operate and release times, and approaches the ideal characteristic, i.e. high resistance when open, and low resistance when closed and it can be mass produced. However, unless it is used with ingenuity, it is difficult to produce an exchange at as low a cost as when using wiper–bank base-metal contacts, but the increased reliability and low maintenance costs may result in its becoming a standard component.

(*c*) From the point of view of fast operation, small size, reliability and long-life, transistor-type devices are attractive for speech-path switching in a telephone exchange. However, as a simple off/on speech-path switching device to be used in a space-division type of exchange, it is not ideal. It does not offer a high off/on resistance ratio; the noise introduced is not negligible, and the parameters vary with temperature. The cost has not been reduced sufficiently for the device to compete economically with those described under (*a*) and (*b*).

Q. **2.** *Describe, with the aid of sketches, the form of construction of the following parts of a reverse-action uniselector:*

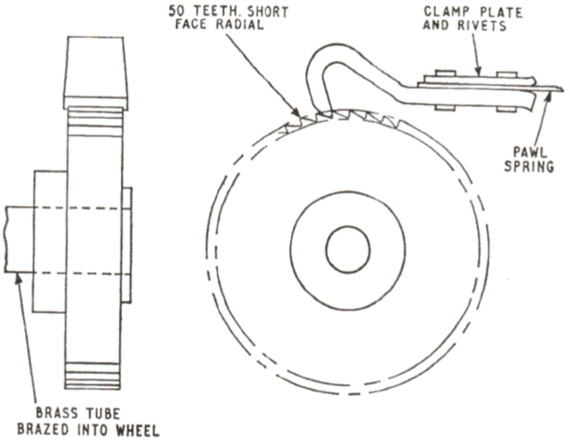
(*a*) *Ratchet and pawl,*
(*b*) *Bank contacts and insulators.*

Of what materials are these parts made and what characteristics of these materials lead to their choice for their respective functions?

A. 2. The construction of the ratchet and pawl of one form of reverse-acting uniselector mechanism is shown in sketch (*a*).

The item described is a Post Office Type-2 uniselector.

The tail of the pawl is attached to the pawl-spring by two rivets in conjunction with a clamp-plate, the end of which, together with the pawl tail, is flared. A slightly convex section is given to the back portion of the pawl where it contacts the pawl back-stop. The tip of the pawl is shaped so that it fully engages a ratchet tooth. The ratchet teeth have the short face placed radially, the tips have a small land



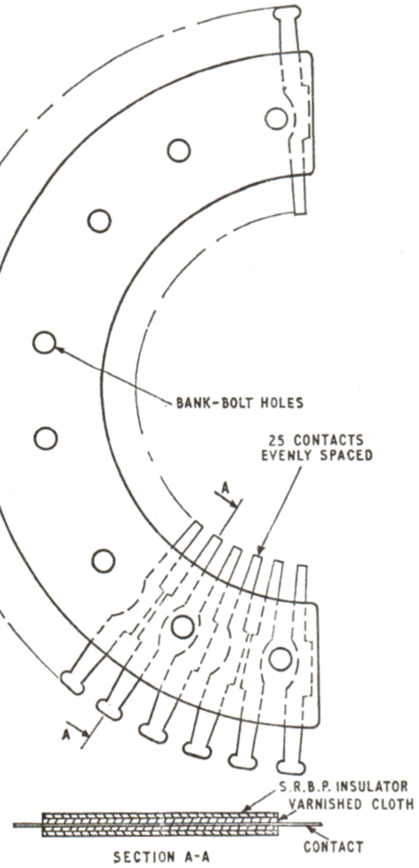
(*a*)

and the roots are radiused. The ratchet wheel is bored to take the wiper hub-tube and the two parts secured together by silver-solder.

The pawl is formed from spring-steel strip. A considerable amount of scuffing action and high pressures occur at the pawl tip and the hardness of spring-steel is essential in preventing rapid wear at this point. The toughness of this material prevents fracture of the pawl during the repetitive shock-loads acting when the uniselector is stepping.

The ratchet wheel is machined from aluminium bronze, an alloy of 10 per cent aluminium and 90 per cent copper. This material is tough and hard-wearing and thus able to withstand the scuffing and impacting action of the pawl. Also, as it would be difficult to obtain a reliable plating on the ratchet teeth, the non-corroding property of aluminium bronze is useful.

(*b*) The construction of the bank-contacts and insulators of a typical uniselector is shown in sketch (*b*).



(*b*)

The contacts are shaped to serve as a wiring tag at the outer end, cut-away to clear the bank-bolts in the centre portion, and have radial sides at the inner end; the end itself is cut square. The contacts are arranged, equally spaced, in an arc between two sectors of tacky, thin varnished cloth, which hold them in position. On either side of the varnished cloth is a sector of hard insulating material which forms the main insulation in the bank. A complete bank is built up of a number of such units with aluminium spacers between them, the whole being clamped between mild-steel frames by several through-bolts.

The contacts may be secured by means other than varnished cloth and modern practice follows one of the methods:

(i) Contacts secured direct to synthetic resin bonded paper (S.R.B.P.) insulators by a thermoset adhesive in which the S.R.B.P. is thicker to compensate for the elimination of the varnished cloth.

(ii) Eyeletted construction—the contacts are secured to a thin sheet of S.R.B.P. by punching a half-shear rivet from them. The main S.R.B.P. sectors then have holes to correspond with the rivet heads.

The contacts are stamped from nickel-silver strip which is an alloy of nickel, copper and zinc. This material has a high resistance to corrosion and will take a high polish, thus ensuring consistently low contact resistance. With reasonably low contact pressures, the alloy is long-wearing. Brass contacts were formerly used for light-duty applications, but this material is inferior to nickel-silver in all respects (except cost).

The main insulation is provided by the sectors of S.R.B.P. which possesses a high surface resistivity, small water-absorption factor and is fairly resistant to tracking. It is formed under very high pressure and is thus virtually incompressible so that dimensional stability of the bank for a long period of service is ensured.

Q. 3. *Describe with the aid of a circuit diagram the operation of a one-v.f. receiver as used for inband signalling.*

In what circumstances might a multi-frequency (5-frequency or 6-frequency) signalling system be used in association with a one-v.f. system? What advantages would this have?

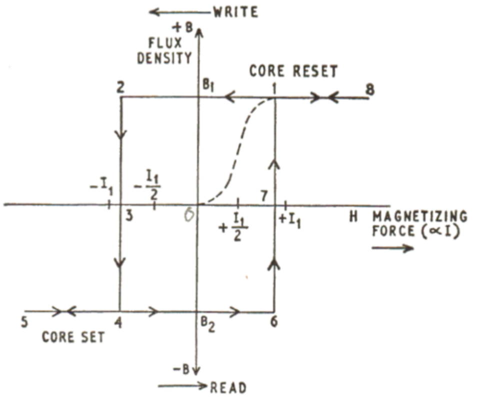
A. 3. For a description of the circuit operation, and a circuit diagram, see A. 7. Telephony C, 1964. *Supplement*, Vol. 58, p. 50, Oct. 1965.

When a call is set up between two common-control type exchanges, it is necessary that the digital information received by the register in one exchange shall be transferred to a register in the other exchange. It is desirable that the information shall be passed quickly, not only to achieve a fast call set-up time, but also to reduce the holding time of register equipment. A 1 v.f. signalling system can be used, employing pulsing of the single signal frequency for the numerical information. For the characteristics of a trunk telephone circuit, there is a practical limit to the signalling speed attained in this way and this is determined by the frequency-attenuation characteristic of the circuit. A multi-frequency (m.f.) code, using a 2 out of 5 (or 6) combination for each digit, can give a faster signalling system between registers using practical and economic equipment. A signalling speed in excess of 100 ms per digit is practicable. Such equipment can be provided in each register, or shared between registers, the line (or supervisory) signalling being handled by 1 v.f. signalling equipment provided on a per line basis.

Q. 4. *Explain, by reference to the magnetization characteristic, the principle of operation of a typical ferrite storage device.*

Draw a simple sketch to show how nine ferrite cores can be wired together to form a 3 × 3 matrix with write and read facilities.

A. 4. Sketch (*a*) shows the magnetic characteristic of a typical ferrite magnetic core used in storage devices.



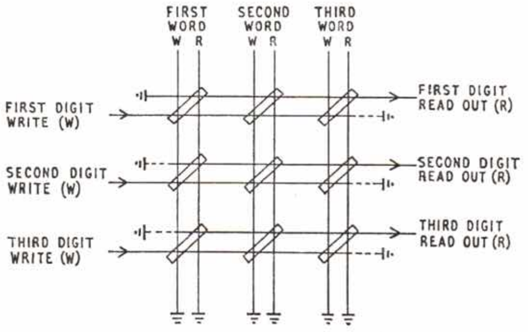
(*a*)

If the core were initially unmagnetized, and subjected to an increasing magnetizing force, $+H$, the magnetic flux density, B, would build up along a curve such as that shown dotted, until a point was reached such that further increments in the value of $+H$ resulted in little further increase in the flux density. This is the saturate value of the flux density, B. If the value of H were now reduced to zero, the core would retain a magnetic field of density B_1. The value of OB_1

on the flux density scale is a measure of the "retentivity" of the core material. The effect of H continuing to change in the same direction and acquiring an increasing negative value would be to reach a point 2, beyond which the flux would rapidly reduce to zero at 3. The negative value of H necessary to reduce the flux to zero is known as the *coercive force* of the material. A further increase in the negative value of H would produce a reverse flux which would finally saturate at point 4, at a value B_2, similar to B_1 but opposite in direction. Further increases in the negative value of H to a point such as 5 would merely maintain the flux at approximately the reverse saturate value B_2. Further excursions of H to a positive value at point 8 would result in the magnetic field changing to a value B_1, traversing the curve B_2, 6, 7, 1 and 8. The core can be said to be "set" in condition B_2, and "reset" in condition B_1.

It is the property of "retentivity" of flux, at values such as B_1 and B_2, which enable the core to remain in one of two stable states "set" or "reset", and so constitute a two-state memory device. To change from one state to the other requires a magnetizing force at least equal to the "coercive force" (03 or 07).

In a practical arrangement used in telephony, two windings are employed on each core for control or "writing-in" purposes. Consider a current $-I_1$ as in sketch (*a*). If the value of $-I_1$ just exceeds the value necessary, in a single winding, to change the core state, then a current of $-I_{1/2}$ (half-write current) will not materially affect the flux when acting alone. However, the coincidence of two such currents acting together in the separate coils will change the state of the core if they are applied in the appropriate sense. A third winding may be used to detect an induced e.m.f. when the state of the core is changed.



(*b*)

Using these principles a core matrix can be assembled in such a way that at each cross-point a core can be set by coincident "half-write" currents on the appropriate row and column. Sketch (*b*) shows a 3 × 3 matrix. Separate wires are shown in each row and column, for writing (W) and reading (R). At an appropriate time, the application of a current $+I_1$ on the column R wire results in the resetting of all cores in the column which have been set, a read-out e.m.f. being indicated on the appropriate row R wires. An e.m.f. of opposite sign will be induced on these wires during the writing-in procedure, but by the use of rectifiers these e.m.f.s can be ignored.

Q. **5.** *Sketch a circuit showing all the items of equipment connected to the P-wire on a local call through a 50-volt 4-digit non-director main exchange employing uniselector calling equipments and two-motion selectors.*

Assuming typical values of resistance for all current sources and zero resistance for the P-wire connexion, calculate the current flow from the final selector to the P-wire holding circuit when a second calling equipment tests this connexion.

Explain the importance of having a low P-wire resistance. What maximum value of resistance would be tolerable?

A. 5. The sketch shows all the items of equipment connected to the P-wire of a local call through a typical 50-volt 4-digit non-director main exchange.

The figure shows the connexions at the moment when the second subscriber's uniselector has stepped on to the engaged outlet, and current is flowing to the uniselector magnet from the final selector earth at point Y. Assuming a zero P-wire resistance between points X, Y and Z, and a nominal 50-volt supply voltage, the total P-wire current can be calculated as follows:

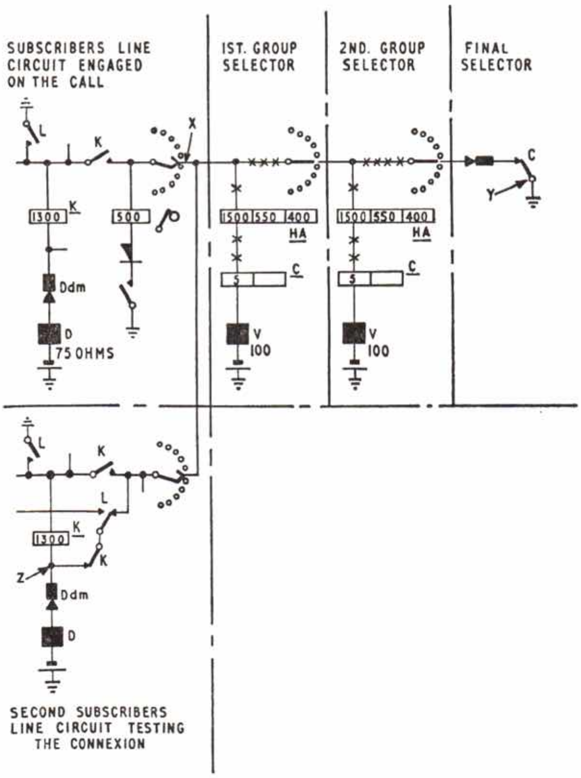
Current in K relay of engaged subscriber's line at $= \dfrac{50}{1375} = 36{\cdot}4$ mA

Current in HA relay of 1st group selector $= \dfrac{50}{1605} = 31{\cdot}2$ mA

Current in HA relay of 2nd group selector $= \dfrac{50}{1605} = 31{\cdot}2$ mA

Current in drive magnet of second uniselector $= \dfrac{50}{75} = 667$ mA

Total P-wire current $= 765{\cdot}5$ mA



Whilst testing this outlet, the K relay of the second subscriber's uniselector is short-circuited, since, assuming zero P-wire resistance, point 2 is at earth potential. It can be seen from the sketch, that if resistance exists between points X, Y and Z, then point X would be at a −ve potential and when the second uniselector was testing, then point 2 would be at a potential more −ve than X. The effect of this could be to give insufficient current for stepping the second uniselector magnet, resulting in the uniselector being arrested on this outlet, or if the P-wire resistance were sufficiently high, the potential of point Z may be sufficient to operate relay K of the second line-circuit resulting in double switching. This illustrates an important aspect of the P-wire resistance. There are similar considerations when a group selector or a final selector tests the connexion, but due to the direct stepping of the uniselector magnet to the P-wire earth, the above is probably the most onerous condition.

Of course there are other considerations which are affected by P-wire resistance. Obviously the P-wire holding relays must have sufficient current, and if the P-wire is used for metering, a sufficient metering current must flow. The double switching possibility, however, is a most important limitation on P-wire resistance. In practice, the acceptable upper design limit for the total P-wire resistance in an automatic exchange is of the order of 10 ohms.

Q. **6.** *Describe, with the aid of a block diagram, the main functions performed by an electro-mechanical subscriber trunk dialling controlling register and an associated relay-type translator.*

Sketch a simplified circuit to show the principle of operation of the relay-type translator.

Q. **7.** *Explain with a circuit sketch how discriminating and digit-absorbing facilities are provided by a discriminating selector repeater (D.S.R.) at a satellite exchange.*

What are the limitations of the D.S.R. compared with the 1st *code selector/A-digit selector/director combination as a means of providing call-routing facilities in a linked-numbering scheme?*

A. 7. The discriminating and digit absorbing elements of a discriminating selector repeater (D.S.R.) are shown in the sketch. The circuit provides for the absorption of one or two digits and for discrimination after the first, second or third digits.

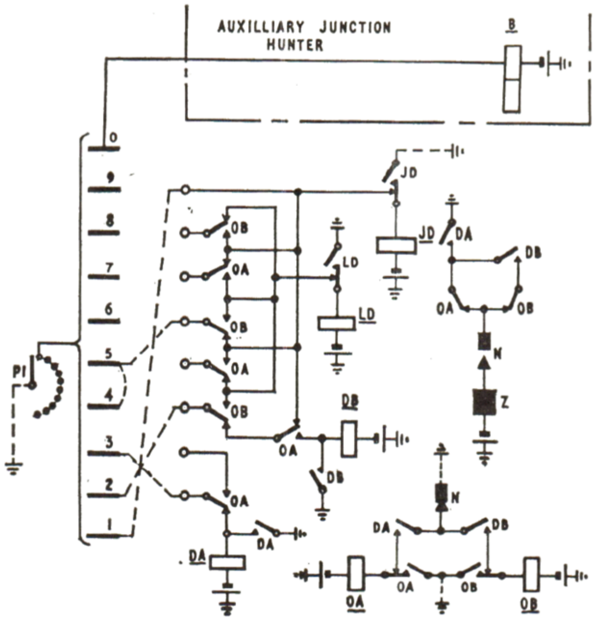
There are six controlling relays, i.e.:

Relays, DA and DB, which release the D.S.R. to absorb first and second digits respectively;

Relays, OA and OB, which rearrange conditions on the P1 marking bank when the D.S.R. returns to normal after absorption of the first and second digits;

Relay, JD, which effects junction discrimination by disconnecting the D.S.R. stepping and switching circuits and routes the call to the main exchange via the D.S.R. junction hunter; and

Relay, LD, which effects local discrimination by releasing the junction to the main exchange and allowing the D.S.R. to function as a group selector.



When a D.S.R. is taken into use, the junction hunter seizes a free junction to the main exchange, consequently the first pulse train received steps both the D.S.R. and a 1st selector at the main exchange to the level corresponding to the digit dialled. At the end of the pulse train, the D.S.R. takes one rotary step and the earth connected to wiper, P1, is extended via the cross-connexion field to a discriminating relay.

The sketch illustrates a typical cross-connexion arrangement for discriminating between local and junction calls, and it is assumed that the numbering range of the local satellite exchange is 32,400–32,599 and that of the main exchange 2xxxx.

For local exchange calls, "3" will be received as the first digit, the D.S.R. and a 1st-selector at the main exchange will step to level 3. At the end of the pulse train, the D.S.R. will take one step into the bank and extend earth from wiper, P1, to operate relay DA. Relay DA, in operating, energizes the release magnet, Z, to release the D.S.R. and operates relay OA, which locks. Receipt of "2" as the second digit steps the D.S.R. and a 2nd-selector at the main exchange to level 2; relay DB operates and this releases the D.S.R. and operates relay OB, which locks. Receipt of "4" or "5" as the third digit steps the D.S.R. and a 3rd-selector at the main exchange to the appropriate level, relay LD operates, releasing the junction to the main exchange and allowing the D.S.R. to function as a 3rd-group selector in the local exchange.

For main exchange calls, "2" will be received as the first digit, the D.S.R. and a 1st-selector at the main exchange will step to level 2 and relay JD will operate and lock. After the operation of relay JD, all subsequent pulse trains are repeated to the main exchange, the D.S.R. remains on the first rotary step of level 2 but is ineffective until it is finally released at the end of the call.

The circuit operation for calls to level "1" services, e.g. "100," is the same as for calls to main exchange numbers, except, of course, that relay JD will be operated on receipt of first digit "1."

For national number dialled calls, "0" will be received as the first digit, the D.S.R. and a 1st-selector at the main exchange will step to level "0" and earth will be extended by wiper, P1, to operate relay B of the auxiliary junction-hunter associated with the D.S.R. in use. The auxiliary junction hunter seizes a trunk circuit to the group switching centre (G.S.C.), which is usually at the main exchange, and extends a pulsing path via a metering-over-junction (M.O.J.) type auto-auto relay set to controlling register translator equipment at the G.S.C. When a trunk circuit is seized by the auxiliary junction hunter, the D.S.R. and local circuit to the main exchange are released.

The limitations of a D.S.R. as a means of providing call routing facilities in a linked-numbering area arise from the lack of routing flexibility possessed by any Strowger non-director system when compared to a register-controlled system. The 1st-code selector/A-digit selector/director combination has code translation facilities which make the routing of a call independent of the digits dialled, this permits flexibility of routing, the insertion of routing digits by the director and in-built pulse regeneration. When the system grows, junctions can be rearranged and new exchanges added in units of up to 10,000 lines without alteration of existing subscribers' numbers.

In a non-director linked-numbering area, all routing digits must be dialled by the caller, and the relationship between digits dialled, selector switching-stages and junction line-plant is closely related. Provision for ample growth results in a high capital cost, and in the early years, operational inefficiency due to the necessity for additional switching-stages, uneconomic junction routes and long dialling-codes. If such provision is not made, however, expansion would be complicated by the need for large-scale number changes and expensive rearrangement of junction plant. These difficulties may be alleviated by the ability of a D.S.R. to discriminate and absorb digits but the basic limitations still remain.

Q. **8.** *Calls from subscribers at exchange A are routed to exchange B over junctions employing loop-disconnect pulsing with direct repetition.*

What factors govern the maximum loop resistance of these junctions when:

(*a*) *all the calls over this junction route terminate at exchange B,*
(*b*) *some calls are switched at exchange B over loop-disconnect junctions to exchange C?*

If in case (b) the A–B junction loop resistance exceeded the maximum permissible value for tandem pulsing what equipment changes might permit the tandem traffic to be routed in this way?

A. **8.** (*a*) When calls originating at exchange A terminate at exchange B, and an auto-auto relay-set with direct pulse repetition is provided at exchange A, the following factors govern the permissible loop resistance of A to B junction:

(*i*) the operate-current sensitivity of the A relays at exchange B,
(*ii*) the operate-current sensitivity, and resistance, of the D and I relays in the auto-auto relay-sets at exchange A,
(*iii*) the type of selectors used at exchange B. With a certain type of circuit, if the line resistance is too high, a premature vertical step may occur on "initial pick up" of the selector due to a long period of relay-contact bunching. Also, distortion of received pulses will occur to an extent which depends upon the various designs of relays used,
(*iv*) whether or not the junction is amplified,
(*v*) the line characteristics, in addition to the loop resistance. Line leakance acts as a resistive shunt across a pulsing relay, reducing its operate time and increasing its release time, i.e. producing positive distortion of pulses. Capacitance of the line will increase the release time of the pulsing relays so giving positive distortion. In addition, it may cause split pulsing due to the presence of a damped oscillation caused by the capacitor having been charged to a high voltage on the opening of the pulsing contacts, and subsequently discharging through the pulsing relay. The positive pulse distortion may be particularly evident over amplified junctions due to the connexion of conductors in parallel to form a phantom circuit for pulsing, in addition to the added capacitors necessary in 2-wire to 4-wire terminations.

(*b*) When some of the calls are switched at exchange B to exchange C via a further loop-disconnect junction, then an auto-auto relay-set will be required at exchange B, and the above considerations will apply to the B to C junction. Further restrictions on the A to B junctions which may be necessary will depend primarily on the type of transmission bridge used at exchange B, i.e. capacitor-type or transformer-type.

If a capacitor-type transmission bridge is used at B it is necessary to restrict the loop resistance of junction AB considerably due to,
(*i*) the effects of longitudinal surges via the bridge capacitors causing interference between adjacent junctions,
(*ii*) difficulties which may occur when the called subscriber answers. If the A relay were operated under limiting conditions, the reversal of polarities which constitutes the answer signal may result in fall out of the A relay due to its failure to reoperate on the reversed flux because this flux is reduced in value due to hysteresis effects in the relay core,
(*iii*) difficulties which may occur following the sending of a pulse train, when an inductive supervisory relay is introduced into the loop at the sending end. This may cause a spurious pulse to be added.

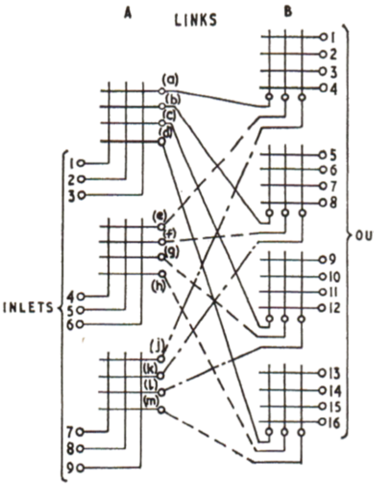
If a transformer-type bridge is used, together with 2-stage drop-back at exchange A, where necessary, this will usually permit the loop resistances of both the A to B and the B to C junctions to be limited only by the performance of the appropriate incoming selectors. This is because the high-impedance pulsing relay may be replaced by a high-speed relay with improved pulsing performance and the fact that the transformer suppresses longitudinal surges during dialling.

If the junction A–B loop resistance exceeded the maximum permissible value for tandem pulsing, the following equipment changes might be made to permit tandem traffic:
(*i*) An electro-mechanical pulse regenerator may be used at B. This would permit similar junction limits for A–B to those imposed by incoming selector equipment, e.g. 2,000 ohms maximum loop resistance.
(*ii*) A signalling system using double-current pulsing may be used. The improved pulse-distortion performance results in limits of up to 8,200 ohms in certain cases.
(*iii*) A v.f. signalling system may be used, employing v.f. pulsing for digital information.
(*iv*) An out-band signalling system may be used over carrier circuits provided with the facility of pulsing the channel carrier frequency.
(*v*) In cases where the speech circuits may be provided via a p.c.m. system, the signalling "bit" (one per channel) may be used for signalling in a variety of ways.

Q. 9. *Illustrate the trunking principle used in crossbar exchanges by drawing a simple sketch to show how any one of 9 trunks can be connected through two switching stages using "3 inlet/4 outlet" switches to any one of 16 trunks. Explain the term blocking in relation to your sketch.*

How can the availability of a crossbar selector be made to exceed the number of selecting bars?

A. 9. The required diagram is shown in sketch (*a*). By the use of 12 links, a–m inclusive, any of the 9 inlets may be connected to any

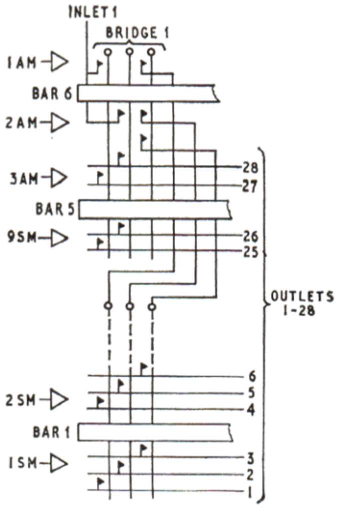


(*a*)

of the 16 outlets. Consider inlet 1 connected to outlet 1. On the top switch A, cross-point 1(*a*) is actuated, extending the connexion to link (*a*). On the top switch B, cross-point 1 (*a*) is actuated, extending the link (*a*) to outlet 1.

Whilst this connexion is held, although outlets 2, 3, 4 may be free, connexion to these outlets from either input 2 or 3 is not possible, as the only connecting link (*a*) is busy. This illustrates the condition known as blocking. Further connexions between the top switch A and the top B switch are blocked due to insufficient provision of link circuits. Such blocking is an inevitable feature of such a trunking arrangement.

One method of increasing the availability of a cross-bar selector is by the use of auxiliary magnets. By the selection of a switch outlet dependent upon the operation of two magnets (select and auxiliary), the availability can be made to exceed the number of selecting bars. Sketch (*b*) shows a 6-bar type of selector, in which 3 of the bar magnets



(*b*)

are used as auxiliary magnets (AM), the remaining 9 being used as select magnets (SM). One inlet, or bridge, is shown. The operation of any 3 AM with any of 9 SM would give 27 combinations, but operation of 3 AM and 9 SM are mutually exclusive events, thus reducing the number to 26. Two more possibilities are used employing AM only, viz. 1 AM + 3 AM, and 2 AM + 3 AM. In this way each inlet of the 6-bar switch can be connected to any of 28 outlets.

Q. 10. *Describe, with a circuit sketch, the operation of the A-digit selector from seizure to the commencement of rotary hunting.*

What circuit arrangements are provided to prevent the selector being held by a line fault which causes a permanent calling condition?

TELECOMMUNICATION PRINCIPLES C, 1967

Students were expected to answer any six questions.

Q. 1. (*a*) *Illustrate by means of sketches the distribution of the electric field within the dielectric of*

(*i*) *a parallel-plate capacitor,*
(*ii*) *a length of coaxial cable.*

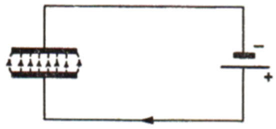
Label the sketches to show the polarity of an applied battery, that of the charge displaced and the direction of the electric flux.

(*b*) *At a certain frequency a capacitor can be represented by a reactance of* 1,000 *ohms in series with a resistance of* 0·1 *ohm. Calculate for this capacitor*

(*i*) *the power factor,*
(*ii*) *the loss angle,*
(*iii*) *the Q-factor,*
(*iv*) *the equivalent shunt resistance.*

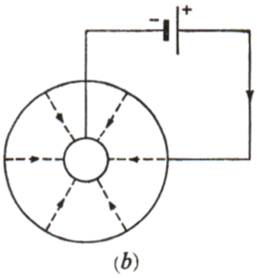
A. 1. Sketch (*a*) and sketch (*b*) apply to the electric field within the dielectric of,

(*a*) a parallel-plate capacitor, and
(*b*) a length of coaxial cable.

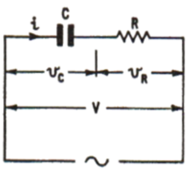


(*a*)

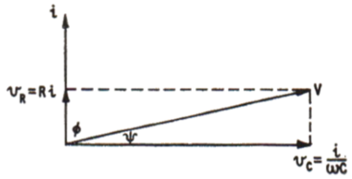
The equivalent series circuit of the capacitor and the vector diagram of the current supplied and the various voltages are given in sketches (*c*) and (*d*).



(*b*)



(*c*)



(*d*)

(*i*) Power factor $= \dfrac{\text{Power Supplied}}{\text{Volt–Amperes}} = \dfrac{(0\cdot 1)i^2}{\sqrt{(0\cdot 1^2 + 1{,}000^2)} \times i^2}$

$$\simeq \frac{0\cdot 1}{1{,}000} = \underline{10^{-4}}$$

(*ii*) If ψ is the loss angle

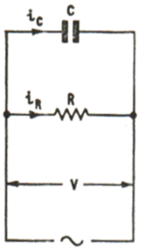
$$\sin\psi = \text{Power factor} = \cos\varphi = 10^{-4},$$

Since $\sin\psi$ is very small.

$$\therefore\quad \psi = \sin\psi = 10^{-4}\text{ radian} = \underline{0\cdot 00573°.}$$

(*iii*) Q-factor $= \dfrac{1}{\omega CR} = \dfrac{1{,}000}{0\cdot 1} = \underline{10^4.}$

(*iv*) The equivalent shunt circuit of the capacitor is shown in sketch (*e*).



(*e*)

$$\text{Admittance} = \frac{1}{R} + j\omega C = \frac{1}{0\cdot 1 - j1{,}000} = \frac{0\cdot 1 + j1{,}000}{0\cdot 1^2 + 1{,}000^2},$$

$$\simeq \frac{0\cdot 1}{1{,}000^2} + \frac{j}{1{,}000},$$

$$\therefore\quad \underline{R = 10\text{ Mohm.}}$$

Q. **2.** *An iron-cored transformer has a primary-to-secondary turns ratio of* 1 : 2, *and the no-load primary inductance is* 0·5 *H. In all other respects the transformer may be considered ideal. Calculate*

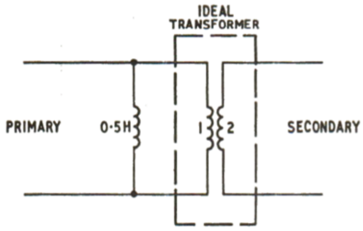
(*a*) *the inductance of the secondary winding with the primary on open-circuit,*

(*b*) *the overall inductance of the two windings connected* (*i*) *in series aiding,* (*ii*) *in series opposing,*

(*c*) *the magnetizing current when a voltage of* 1 *volt r.m.s. at a frequency of* 800 *Hz is applied to the primary winding,*

(*d*) *the total primary current when, under the conditions of* (*c*), *a resistance of* 10 *kohm is connected to the secondary winding.*

A. 2. The equivalent circuit of the transformer is given in the sketch.



(*a*) The inductance measured across the secondary terminals will be

$$n^2L_p = 2^2 \times 0\cdot 5 = \underline{2\cdot 0\text{ H.}}$$

i.e. the reflected (shunt) inductance of the primary winding.

Because the transformer can be considered ideal in all other respects, the inherent (shunt) self inductance of the secondary winding can be assumed to be infinite.

(*b*) When the two windings are connected in series, they constitute an autotransformer having a turns ratio of

(*i*) 1 : 3 in series-aiding,
(*ii*) 1 : 1 in series-opposing,

with a load of 0·5 H across the primary winding.

Thus, the overall inductance will be:

$\underline{3^2 \times 0\cdot 5 = 4\cdot 5\text{ H in the series-aiding connexion.}}$

$\underline{1 \times 0\cdot 5 = 0\cdot 5\text{ H in the series-opposing connexion.}}$

(*c*) Magnetizing current $= \dfrac{1}{j2\pi \times 800 \times 0\cdot 5} \simeq \underline{-\,j0\cdot 4\text{ mA.}}$

(*d*) When a load of 10 kohms is connected across the secondary winding, the equivalent primary circuit is an inductance of 0·5 H in parallel with a resistance of $\dfrac{10{,}000}{2^2} = 2{,}500$ ohms.

$$\text{Current taken by this resistance} = \frac{1}{2{,}500} = 0\cdot 4\text{ mA.}$$

$$\text{Total primary current} = 0\cdot 4 - j0\cdot 4,$$

$$= \underline{0\cdot 5656 \,\overline{\diagdown 45°}\text{ mA.}}$$

Q. **3.** *State Thevenin's Theorem.*

A signal generator has its output terminals connected to a variable resistor in parallel with a valve voltmeter. The indication of the voltmeter as the resistance is varied is shown in Table 1. *Determine the e.m.f. of the generator and the internal resistance.*

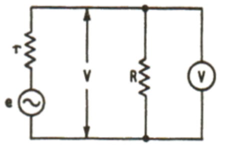
TABLE 1

R ohms	$V(\mu V)$
25	33
50	50
100	67

What is the maximum power available from this generator?

A. 3. *Thevenin's Theorem.* The current in the load impedance connected to the output terminals of a network is the same as if this load impedance were connected across a generator which has an e.m.f. equal to the open-circuit voltage measured across the output terminals of the network and an internal impedance equal to the impedance measured across the output terminals with all generators replaced by their internal impedances.

The output circuit of the signal generator can be represented as in the sketch.



From this sketch $\quad V = \dfrac{R}{R + r}e$

$$\text{or,}\quad e = V + \frac{R}{r}V,\ \text{or}\ e - V = \frac{r}{R}V.$$

$$\text{Thus}\qquad e - 33 = \frac{r}{25}0\cdot 33 \quad .\ .\ .\ .\ (i).$$

$$e - 50 = \frac{r}{50}0\cdot 50 \quad .\ .\ .\ .\ (ii).$$

$$e - 67 = \frac{r}{100}0\cdot 67 \quad .\ .\ .\ .\ (iii).$$

$$\text{From }(i)\text{ and }(ii)\quad 17 = 0\cdot 32r\ \text{ or }\ r = \frac{17}{0\cdot 32} = 53\cdot 1.$$

$$(ii)\text{ and }(iii)\quad 17 = 0\cdot 33r\ \text{ or }\ r = \frac{17}{0\cdot 33} = 51\cdot 5.$$

$$(i)\text{ and }(iii)\quad 34 = 0\cdot 65r\ \text{ or }\ r = \frac{34}{0\cdot 65} = 52\cdot 3.$$

$$\text{Let}\quad r = 52\cdot 3.$$

$$\text{From }(i)\qquad e = 33 + \frac{52\cdot 3}{25}0\cdot 33\ \text{ or }\ e = 102\ \mu\text{V}.$$

$$(ii)\qquad e = 50 + \frac{52\cdot 3}{50}0\cdot 50\ \text{ or }\ e = 102\cdot 3\ \mu\text{V}.$$

$$(iii)\qquad e = 67 + \frac{52\cdot 3}{100}0\cdot 67\ \text{ or }\ e = 102\ \mu\text{V}.$$

Thus, $e = 102\ \mu$V and $r = 52\cdot 3$ ohms.

The power supplied by the generator

$$= i^2R = \frac{e^2}{(R + r)^2}R = \frac{e^2}{R + 2r + \dfrac{r^2}{R}}.$$

The power supplied is a maximum when $R = r$, i.e. when it is equal to $\frac{e^2}{4r}$.

$$\therefore \quad \text{Maximum power supplied} = \frac{102^2}{4 \times 52{\cdot}3} \simeq \underline{50\ \mu\text{Watts.}}$$

Q. 4. *The frequency of the generator in Fig. 1 can be varied. Calculate*

(a) the frequency at which the current is a maximum,
(b) the maximum value of the current,
(c) the capacitor-voltage at resonance.

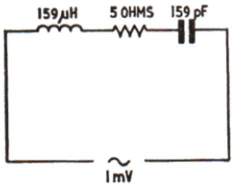


Fig. 1

Sketch a graph to show the variation of the current as the frequency is varied and calculate the value of the current at frequencies 0·25 per cent above and below the resonant value.

A. 4. (*a*) The current is maximum when the reactance of the inductance is equal in magnitude and opposite in sign to the reactance of the capacitance.

$$\text{i.e. when} \quad \omega_0 \times 159 \times 10^{-6} = \frac{10^{12}}{\omega_0 \times 159}.$$

$$\omega_0 = 2\pi f_0 = \frac{10^9}{159} \text{ and } f_0 = \underline{10^6\ \text{Hz.}}$$

(*b*) The maximum value of the current is given by $\frac{\text{voltage}}{\text{resistance}}$.

$$\therefore \quad i = 1/5 = \underline{0{\cdot}2\ \text{mA.}}$$

(*c*) Voltage across the capacitance at resonance is given by

$$\frac{i}{\omega_0 C} = \frac{10^{12}}{10^9} \times 0{\cdot}2 = \underline{200\ \text{mV.}}$$

Let the magnitude of the current at an angular frequency of $\omega_0(1 + \delta)$ radians/sec be i. Since δ is small the value of i at an angular frequency of $\omega_0(1 - \delta)$ will be the same.

$$|i| = \frac{1}{\sqrt{R^2 + \left\{\omega_0(1+\delta)L - \frac{1}{\omega_0(1+\delta)C}\right\}^2}},$$

$$\simeq \frac{1}{\sqrt{R^2 + \left\{\omega_0 L + \omega_0 L\delta - \frac{1-\delta}{\omega_0 C}\right\}^2}},$$

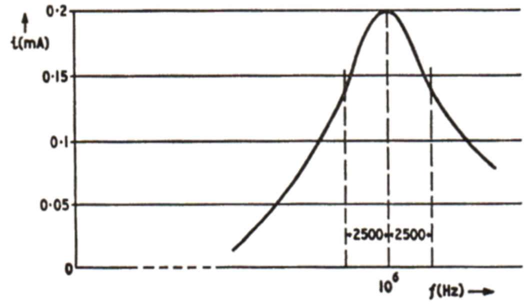
Since $(1 + \delta)^{-1} = 1 - \delta$ from the binomial series (since δ^2 and succeeding terms are negligible).

$$= \frac{1}{\sqrt{R^2 + \delta^2\left\{\omega_0 L + \frac{1}{\omega_0 C}\right\}^2}}, \text{ since } \omega_0 L - \frac{1}{\omega_0 C} = 0.$$

$$= \frac{1}{\sqrt{R^2 + 4\delta^2\omega_0{}^2 L}},$$

$$= \frac{1}{\sqrt{25 + 4\left(\frac{0{\cdot}25}{100}\right)^2 \frac{10^{18}}{(159)^2}(159 \times 10^{-6})^2}},$$

$$= \frac{1}{\sqrt{25 + 25}} = \frac{1}{5}\frac{1}{\sqrt{2}} = \underline{0{\cdot}1414\ \text{mA.}}$$



The current/frequency graph is shown in the sketch.

Q. 5. *The instantaneous current in the aerial of a transmitter is represented by* $I = 100 \sin \pi 10^6 t$

$$+ 40 \sin (\pi 10^6 + 5{,}000)t$$
$$+ 40 \sin (\pi 10^6 - 5{,}000)t \text{ amperes.}$$

Draw and explain a phasor (vector) diagram corresponding to this equation and deduce

(a) the type of modulation,
(b) the frequency of the carrier wave,
(c) the waveform of the modulating signal,
(d) the frequency of the modulating signal,
(e) the modulation depth,
(f) the peak instantaneous value of the current,
(g) the r.m.s. value of the current.

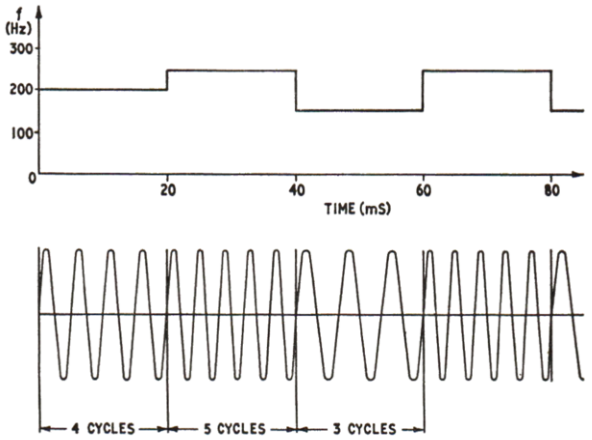
Q. 6. *A carrier wave of frequency 200 Hz is frequency modulated by a signal of square waveform with a full period of 40 ms so that the maximum frequency deviations are ±50 Hz.*

Sketch the waveform of the modulated carrier taking care to draw the correct numbers of cycles corresponding to the signal conditions.

The waveform of the signal is altered from square to sinusoidal with the same peak value and period.

Write down an expression for the instantaneous frequency of the carrier modulated by this sine-wave and calculate the modulation index.

A. 6. The waveform of the modulated carrier is shown in the sketch. In a 20 ms half-period of the modulation there are 5 cycles at the upper frequency of 250 Hz and in the next 20 ms half-period of the modulation there are 3 cycles at the lower frequency of 150 Hz, and so on.



The required expression for the instantaneous frequency, f, Hz of a carrier f_m Hz modulated with a sinusoidal signal of frequency ϕ is given by

$$= f_m(1 + \delta \sin 2\pi\phi t),$$

where, δ is the modulation index.

With the information given

$$f = 200\left(1 + \frac{50}{200} \sin 2\pi 25t\right) \text{Hz.}$$

The modulation index is $\frac{50}{200} = \underline{0{\cdot}25.}$

Q. 7. *Fig. 2 shows a small-signal equivalent circuit of a transistor with the values of the hybrid parameters appropriate to the common-emitter connexion.*

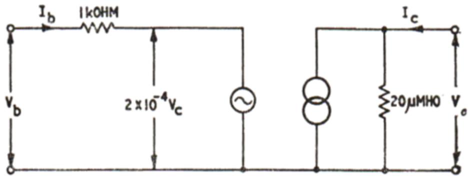


Fig. 2

Assuming that the bias conditions remain unchanged throughout, calculate

(a) the input resistance and the current gain with the collector load on short-circuit,
(b) the voltage gain and the power gain with the collector-load a resistance of 50 kohms.

A. 7. From the information given,

(*a*) h_{11} = Input resistance when output is short-circuited = $\underline{1{,}000 \text{ ohms.}}$
h_{22} = Output admittance with input on open circuit = $20\ \mu$ mhos
h_{12} = Reverse feedback ratio = 2×10^{-4}
h_{21} = Current amplification, or gain, with output short-circuited = $\underline{50.}$

(*b*) Voltage Gain = $\underline{1{,}666 \cdot 6}$

Power Gain = $\underline{4 \cdot \dot{1}6 \times 10^4}$.

***Q.* 8.** *What is the Miller Effect and how can it be reduced?*

Show that the effective input capacitance at the grid of a resistance-loaded triode amplifying stage is given by

$$C_{gk} + C_{ga}(1 + m)$$

where

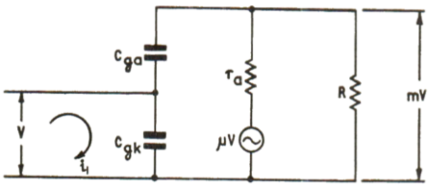
C_{gk} *is the grid-cathode interelectrode capacitance*
C_{ga} *is the grid-anode interelectrode capacitance*
m is the magnitude of the stage gain.

When the stage gain is 8 the effective capacitance is 15 pF and rises to 27 pF when the gain is doubled. Calculate the two valve capacitances.

A. 8. The Miller Effect is the tendency of a triode valve to self-oscillation at high frequencies due to its grid-anode capacitance, which also increases the effective grid-cathode capacitance.

It can be reduced by inserting between the control grid and anode of the valve a second or screen grid which is effectively earthed at high frequencies.

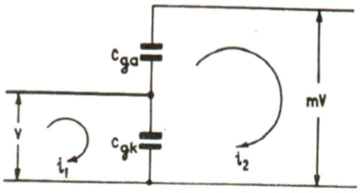
The equivalent circuit of a triode with a resistive load including the grid-anode and grid-cathode capacitances is given in Sketch (*a*).



(*a*)

If the input voltage is V, the output voltage will have a magnitude of mV but will be opposite in phase to V.

For calculating the effective grid-cathode capacitance the circuit can be simplified as shown in Sketch (*b*).



(*b*)

From this
$$V = \frac{i - i_1}{j\omega C_{gk}},$$

and
$$\frac{i - i_1}{j\omega C_{gk}} = -mV + \frac{i_1}{j\omega C_{ga}},$$

$$(i - i_1)\left\{\frac{1}{j\omega C_{gk}} + \frac{1}{j\omega C_{ga}}\right\} = -mV + \frac{i}{j\omega C_{ga}}$$

$$V\left\{1 + m + \frac{C_{gk}}{C_{ga}}\right\} = \frac{i}{j\omega C_{ga}}$$

$$\frac{i}{v} = j\omega C_{gk} + (1 + m)j\omega C_{ga}.$$

Effective input capacitance $= C_{gk} + C_{ga}(1 + m).$

Substituting, $15 = C_{gk} + C_{ga}(1 + 8),$

$27 = C_{gk} + C_{ga}(1 + 16).$

$\therefore \quad C_{ga} = \frac{12}{8} = \underline{1 \cdot 5 \text{ pF,}}$

and $\quad C_{gk} = \underline{1 \cdot 5 \text{ pF}}.$

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